

Mathematical Reflective Thinking Ability in Problem Solving: A Learning Style Perspective

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Received: May 25, 2026 | **Revised:** July 22, 2026 | **Accepted:** August 17, 2026 | **Published:** August 28, 2026

ABSTRACT Mathematical reflective thinking ability is an important higher-order thinking skill that supports students in solving mathematical problems meaningfully. However, students' reflective thinking ability in mathematics is still relatively low and has not been fully developed in mathematics learning. One factor that may influence this ability is students' learning styles. Therefore, this study aimed to describe students' mathematical reflective thinking ability in solving circle problems viewed from the perspective of learning styles. This study employed a qualitative case study approach. The subjects were three eleventh-grade students at a private senior high school in Bandung Regency, representing the visual, auditory, and kinesthetic learning styles. The instruments used were a mathematical reflective thinking ability test, a learning style questionnaire, and interview guidelines. Data was analyzed through triangulation techniques. The results showed that students with an auditory learning style fulfilled four indicators of mathematical reflective thinking ability, kinesthetic students fulfilled three indicators, and visual students fulfilled two indicators. All students were able to identify mathematical concepts and distinguish relevant from irrelevant data in problem-solving. However, all learning style groups failed to fulfill the indicator of generalizing accompanied by reasoning or explanation. These findings provide insights into students' mathematical reflective thinking across different learning styles and may inform the design of instructional strategies that better support the development of reflective thinking in mathematics learning.

Keywords: mathematical reflective thinking ability, mathematical problems, learning style

INTRODUCTION

Learning aims to humanize people. It is because learning plays a crucial role in shaping individual behavior, both in thinking and in action (Halean et al., 2021). In line with this, mathematics is one of the fields that plays a vital role in the learning process. Through mathematics education, students are trained to think systematically, carefully, and accurately, enabling them to meet practical needs and solve various problems in daily life. According to Thahir et al. (2019), mathematics contributes to the development of various thinking and problem-solving abilities. Recent studies also emphasize that

mathematics learning contributes to the development of students' problem-solving competencies needed in 21st century education (Widodo et al., 2023). Furthermore, mathematics learning activities should encourage students to reason, communicate and solve problems meaningfully in various mathematical situations. Therefore, mathematics learning should not only focus on mastering concepts and procedures but also on developing students' higher order thinking skills.

Reflective thinking falls under the category of higher-order thinking ability (King et al., 1997). Reflective thinking was first defined as an active, persistent, and careful process of considering a belief based on the knowledge supporting it, ultimately leading to a conclusion (Dewey, 1933). This also aligns with Balta (2018) who emphasized that reflective thinking also supports students in evaluating strategies and making appropriate mathematical decisions during problem-solving activities. Therefore, mathematical reflective thinking ability is closely related to the mathematical problem-solving process, as a person's reflective thinking can be identified through the problems they solve.

Mathematical reflective thinking ability is of great importance; among other things, they can help students learn to think quickly when developing appropriate problem-solving strategies (Muntazhimah et al., 2021). Mathematical reflective thinking can also serve as a bridge connecting prior knowledge to the problem-solving situations being faced (Sholikhin et al., 2021). Furthermore, Salido & Dasari (2019) state that reflective thinking can make teaching and learning activities more meaningful, thereby helping students construct knowledge that is long-lasting and easily recalled. Furthermore, reflective thinking significantly contributes to students' self-efficacy, conceptual understanding, and mathematical reasoning ability. Hence, mathematical reflective thinking ability plays an important role in supporting students' mathematical problem-solving skills.

The urgency regarding students' mathematical reflective thinking abilities is at odds with the reality on the ground, where these abilities, particularly among high school students, remain low or have not yet developed optimally (Chairunnisa, 2021; Ramadhani & Juandi, 2020; Ratnaningsih & Hidayat, 2020; Zhang & Dempsey, 2019). Field survey results indicate that students' reflective thinking abilities in mathematics are rarely put into practice in problem-solving (Hendriana et al., 2019). One common cause is that teachers still rarely introduce mathematical reflective thinking in the classroom. Teachers often merely provide mathematical formulas and concepts without actively involving students in the thinking process (Salido & Dasari, 2019). Teachers often focus solely on students' final answers, without considering the thought processes they use to arrive at those answers (Fuady, 2017). Consequently, students' mathematical reflective thinking abilities are not optimally developed, limiting their capacity to critically evaluate and refine their problem-solving processes.

One-way teachers help students develop their thinking ability by understanding their learning styles (Jaenudin et al., 2017). It is because learning styles focus on how individuals assimilate and process the information they receive (Costa et al., 2020). The way students express their thoughts and communicate the results of their thinking depends on how they understand and process information (Kartono et al., 2019), who noted that students' difficulties engaging in reflective thinking during learning are partly influenced by their learning styles.

Studies on students' mathematical reflective thinking ability based on visual, auditory, and kinesthetic learning styles have been widely conducted. However, research examining this ability across four types of mathematical problems, namely, closed problems and open-ended problems in both mathematical and real-life contexts remain limited. This limitation has resulted in an incomplete understanding of the characteristics of students' mathematical reflective thinking across different learning styles. Therefore, this study aims to describe the mathematical reflective thinking ability of students with visual, auditory, and kinesthetic learning styles in solving four types of mathematical problems. Based on the problem background outlined above, the objective of this study is to describe students' mathematical reflective thinking abilities in solving mathematical problems from the perspective of learning styles.

RESEARCH METHOD

Based on the objectives of this study, the research approach was qualitative, employing the case study method. According to Creswell (2009), there are five methods of analysis in qualitative research, including the case study. The researcher chose the case study to examine, in greater detail, events, activities, or processes involving one or more individuals. This study was conducted at a private high school in Bandung Regency, West Java Province. The subjects in this study were three 11th-grade students. Initially, all students in the class completed a learning style questionnaire to identify their dominant learning styles. Based on the questionnaire results, one student representing each learning style (visual, auditory, and kinesthetic) was selected after consultation with the mathematics teacher. The selected participants had moderate to high mathematical ability, completed the topic of circles, were able to communicate their thinking clearly during interviews, and agreed to participate in the study. These criteria were applied to facilitate an in-depth exploration of students' mathematical reflective thinking.

Since mathematical reflective thinking is categorized as a higher-order thinking skill, participants were selected from students with moderate to high mathematical ability to ensure that they could express their reflective thinking processes during problem solving. Therefore, the researcher selected students from a class whose mathematical abilities ranged from intermediate to advanced.

In this study, the data were analyzed qualitatively using the Miles and Huberman model, including data reduction, data display, and conclusion drawing. The data were analyzed based on the indicators and verified through triangulation. This triangulation technique involves both test-based and non-test-based methods. Test-based techniques were used to assess students' reflective thinking in mathematics by administering tests that included indicators of this ability. The indicators of mathematical reflective thinking used in this study are as follows (Sumarmo, 2016).

Table 1. Indicators of Mathematical Reflective Thinking Ability

Question Number	Indicators of Mathematical Reflective Thinking Ability
1	Identifying concepts or principles contained in mathematical problems or solution processes, accompanied by reasoning
2	Verifying the validity of arguments, questions, or solution processes
3	Identifying relevant and irrelevant data in problem-solving
4	Drawing analogies accompanied by reasoning and explanations
5	Generalizing accompanied by reasoning or explanations

The non-test methods used in this study consisted of questionnaires and interviews. Questionnaires were used to collect data on students' preferred learning styles. In contrast, interviews were used to obtain more in-depth information regarding students' mathematical reflective thinking ability and learning style preferences. The indicators for each learning style used in this study are as follows.

Table 2. Indicators of Learning Styles

Types of Learning Styles	Indicators
Visual	- Using all available visual teaching materials, such as graphs, maps, notes, videos, and so on. - Writing things down to help remember them. - Visualizing images, words, and concepts in one's mind. - Paying attention to something interesting.
Auditory	- Using a voice recorder as a supplementary teaching tool. - Gathering information by listening. - Memorizing something by saying it aloud. - Listening to what the speaker says during a group discussion.
Kinesthetic	- Remembering a word through movement. - Writing down the key points of the lesson several times to understand the concept. - Demonstrating the material in front of the class. - Engaging in any physical activity during a group discussion.

RESULTS AND DISCUSSION

This study aims to describe students' mathematical reflective thinking abilities in solving mathematical problems from the perspective of learning styles, namely visual, auditory and kinesthetic. Differences in learning styles may influence students' reflective thinking processes in solving mathematical problems (Maulida et al., 2024). Therefore, the presentation of the results and discussion is organized based on each learning style to describe students' reflective thinking characteristics in mathematical problem solving. To facilitate cross-case comparison, Table 3 summarizes the mathematical reflective thinking indicators achieved by each participant before the findings are discussed in detail.

Table 3. Cross-case Summary of Indicators

Indicator	Visual (VS)	Auditory (AS)	Kinesthetic (KS)
Indicator 1	✓	✓	✓
Indicator 2	×	✓	✓
Indicator 3	✓	✓	✓
Indicator 4	×	✓	×
Indicator 5	×	×	×

Subject with a Visual Learning Style

The visual participant (VS) requires visual aids when learning. This is in line with DePorter & Hernacki (2008), visual learners tend to understand information better through written symbols, diagrams, and visual representations. Additionally, VS like to take notes because they use these notes to review the material. VS finds it easier to understand the material when the teacher writes important points on the blackboard and supplements them with verbal explanations; this is especially true in mathematics, where formulas or example problems need to be written on the blackboard. Furthermore, during interviews, VS demonstrated several characteristics, such as difficulty expressing their thoughts and a tendency not to be easily distracted by their surroundings while studying.

The following are the results of the mathematical reflective thinking ability test for students with a visual learning style in solving problems on the first indicator shown in Figure 1.

Untuk menentukan jari-jari lingkaran, di hanya dapat digunakan untuk lingkaran besar, sementara untuk lingkaran kecil, dibutuhkan informasi tambahan seperti jari-jari, koordinasi, dll, mengenai lingkaran kecil.

English translation: To determine the radius of the circle, the available information can only be used for the large circle. For the small circle, additional information is required, such as its radius and coordinates.

Figure 1. Visual subject response to the 1st indicator

Figure 1 shows that VS provided the correct answer. Based on the written answers and interview results, the subject identified the given and unknown data in the problem and the mathematical concepts/principles contained in it, enabling the subject to provide reasons relevant to the problem. This situation aligns with the findings of Kartono et al. (2019), who reported that students with a visual learning style can identify the mathematical concepts involved in a non-trivial mathematical problem. Although students' answers tend to be brief, visual learners can still understand the problem well. This situation aligns with research by Nabilah et al. (2022), which states that visual learners tend not to write down known and asked information, yet can articulate it during interviews. Additionally, this is supported by the findings of Amaliyah et al. (2021), who noted that a characteristic of students with a visual learning style is providing brief answers to each question posed by the researcher. Students with a visual learning style are not particularly adept at communicating the results of their thinking. This situation aligns with the characteristics of individuals with a visual learning style, who know what to say but cannot express it in words (DePorter & Hernacki, 2008). Thus, it can be concluded that the subjects met the first indicator. The following is a description of VS's mathematical reflective thinking ability in solving mathematical problems for the second indicator, as shown in Figure 2.

2) Pernyataan anggota TNI tersebut benar, karena dinyatakan bahwa radar milik TNI hanya dapat mendeteksi benda asing sejauh 10 km. Sementara benda asing terletak sejauh 11 km dari titik pusat radar.

English translation: The TNI member's statement is correct because the TNI radar can only detect a foreign object up to 10 km away, whereas the foreign object is located 11 km from the radar's center.

Figure 2. Visual subject response to the 2nd indicator

Figure 2 shows that VS was unable to provide the correct answer, as indicated by the use of an inappropriate argument. This finding contradicts the results of a study by Jaenudin et al. (2017), which stated that all visual learners are already capable of verifying the validity of an argument based on the concepts or properties used. Based on the answers and interview results, it is evident that VS overlook certain statements in the questions, preventing them from identifying the mathematical concepts embedded in the given problems. Consequently, they are unable to verify the validity of the

statements presented in the questions because they do not recall the correct problem-solving procedures. In line with this, Nabilah et al. (2022) found that students with a visual learning style tend to be careless in verifying the truth of a statement, leading to incorrect answers. Thus, it can be concluded that the student has not met the second indicator. The following is a description of the student's mathematical reflective thinking ability in solving mathematical problems for the third indicator, as shown in Figure 3.

3) a. percobaan I	percobaan II	percobaan III
$x^2 + y^2 = 289$	$x^2 + y^2 = 289$	$x^2 + y^2 = 289$
$(-12)^2 + (5)^2 = 289$	$(17)^2 + (-9)^2 = 289$	$(8)^2 + (15)^2 = 289$
$144 + 25 = 289$	$289 + 81 = 289$	$64 + 225 = 289$
$169 \neq 289$	$370 > 289$	$289 = 289$
masuk	tidak masuk	tepat di antara zona kuning dan biru

English translation: Trial I: inside. Trial II: outside. Trial III: exactly between the yellow and blue zones.

Figure 3. Visual subject response to the 3rd indicator

Figure 3 shows that VS can already identify relevant and irrelevant data. The subject appears capable of identifying the problem and determining the conditions required to solve it. However, on the provided answer sheet, he only wrote his calculations, not the conclusion derived from them. This situation aligns with the research by Jaenudin et al. (2017), which found that visual learners can identify relevant and irrelevant data by first performing calculations. Additionally, according to Nabilah et al. (2022), visual learners can state conclusions but do not write them on the answer sheet when answering questions. Based on the interview results, they were able to explain their answers better than what was written in Figure 3. Thus, it can be said that the students have met the third indicator. The following is a description of the students' mathematical reflective thinking ability in solving mathematical problems for the fourth indicator, as shown in Figure 4.

4) a. $AB > r_1 + r_2$ / $AB < r_1 + r_2$	$r_1 + r_2$
$\sqrt{x_2^2 - x_1^2 + y_2^2 - y_1^2}$	$\sqrt{1600 + 2500}$
$\sqrt{25 + 15^2 + (-17)^2 + 13^2}$	$\sqrt{1600 + 2500}$
$\sqrt{1600 + 3600}$	$24 + 24$
$\sqrt{5200}$	48
$\sqrt{5200} \approx 72$	$AB > r_1 + r_2$
≈ 72	Saling lepas
b. Sepeda motor, pengelasan = sudah habis waktu	

English translation: The circles are disjoint. Motorcycle; explanation: time ran out.

Figure 4. Visual subject response to the 4th indicator

Figure 4 shows that VS is not yet able to draw analogies accompanied by reasoning and explanations. The subject appears capable of identifying and determining the correct conclusion from the given problem. Additionally, the subject is not yet able to provide examples similar to the given case. According to Kartono et al. (2019), visual learners are unable to draw analogies because they do not understand the given case. Based on the interview results, VS knew what conditions needed to be met, for the example he provided to be similar to the case in the problem. However, he ran out of time while attempting to provide mathematical proof for the example he provided. Thus, it can be said that VS has not yet met the fourth indicator. The following is a description of VS's mathematical reflective thinking ability in solving mathematical problems for the fifth indicator, as shown in Figure 5.

g) a. $x^2 + y^2 = r^2$	$x^2 + y^2 = r^2$	$x^2 + y^2 = 100$
$x \cdot x_1 + y \cdot y_1 = r^2$	$2x + y = 10$	$6^2 + 8^2 = 100$ titik = (6, 8)
$x \cdot 3 + y \cdot 4 = 25$	$4x + 2y = 20$	$36 + 64 = 100$
$3x + 4y = 25$	(4, 2)	$100 = 100$
— — — — —	— — — — —	— — — — —
$x^2 + y^2 = r^2$		$x \cdot x_1 + y \cdot y_1 = r^2$
		$x \cdot 6 + y \cdot 8 = 100$
		$6x + 8y = 100$
		1
b. Untuk menentukan persamaan garis singgung, hanya perlu memasukkan titik singgung ke dalam persamaan lingkaran		

English translation: To determine the tangent-line equation, substitute the point of tangency into the circle equation.

Figure 5. Visual subject response to the 5th indicator

Figure 5 shows that VS is not yet able to generalize, provide reasoning, or offer explanations. This situation is consistent with the findings of Jaenudin et al. (2017), who stated that students with a visual learning style are not yet able to generalize their reasoning. VS appears to have been able to complete the given table correctly, but the subject applied the equation of the tangent line through a point on the circle with center at O . The process of deriving the equation of the tangent line was not evident on the provided answer sheet. Based on the interview results, it appears that the subject's knowledge consists solely of rote memorization, so the subject cannot provide an appropriate justification for the generalization obtained. Pamungkas et al. (2018) indicate that students with a visual learning style can perform correct and complete calculations but are not yet able to provide generalizations. Thus, it can be concluded that SV has not yet met the fifth indicator.

Subject with an Auditory Learning Style

The auditory participant (AS) tends to rely more on the teacher's explanations to understand the material. This need is particularly helpful when SA is first learning the material. After the teacher explains it, AS prefers to hone their ability through practice problems. AS also stated, that they only need images, depending on the specific material. AS requires a quiet environment for understanding the material, and their method of retaining it involves explaining it verbally to their peers. This finding aligns with the characteristics of auditory learners who enjoy discussion and verbal communication during learning activities (Sanggorani et al., 2024). The following are the results of the mathematical reflective thinking ability test for students with an auditory learning style in solving problems on the first indicator, as shown in Figure 6.

Left page calculations:

$$\begin{aligned} \text{Jwb. b. } r \text{ jari } r \text{ lingkaran besar} &= 7 \text{ cm} \\ \text{tp lingkaran besar} &= 7,7 \\ \text{luas lingkaran besar} &= \pi \cdot R^2 \\ &= 22 \cdot 7^2 \\ &= 22 \cdot 49 \\ &= 1078 \\ &= 1078 \text{ cm}^2 \\ \text{Diameter lingkaran besar} &= 14 \text{ cm} \\ \text{keliling lingkaran besar} &= 2 \pi \cdot R \\ &= 2 \cdot 22 \cdot 7 \\ &= 44 \text{ cm} \\ \text{Persamaan lingkaran besar} &= (x-7)^2 + (y-7)^2 = 49 \\ &= (x-7)^2 + (y-7)^2 = 49 \\ &= x^2 - 14x + 49 + y^2 - 14y + 49 = 49 \\ &= x^2 + y^2 - 14x - 14y + 49 = 0 \end{aligned}$$

Right page calculations:

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-14)^2 - 4(-14)(49) \\ &= 196 + 686 \\ &= 882 \\ x^2 + y^2 - 14x - 14y + 49 &= 56 \\ 7^2 + y^2 - 14(7) - 14(7) + 49 &= 56 \\ 49 + y^2 - 98 - 98 + 49 &= 56 \\ y^2 - 98 - 98 + 49 &= 56 \\ y^2 - 147 &= 56 \\ y^2 &= 203 \end{aligned}$$

lingkaran kecil tidak bisa dicari karena data mengenai persinggungan lingkaran kecil dengan titik pusat lingkaran besar tidak diberikan

English translation: Left image: the radius of the large circle is 7 cm; its center is (7, 7); its area, diameter, circumference, and equation are then calculated. Right image: the small circle cannot be determined because information concerning its tangency to the center of the large circle was not provided.

Figure 6. Auditory subject response to the 1st indicator

Figure 6 shows that AS provided a reasonably accurate explanation. The subject first wrote down the radius, circumference, area, and equation of the larger circle. However, there was an error in the equation provided. In addition, the subject calculated the discriminant of the circle even though the purpose of doing so was not apparent. Based on the interview results, AS was able to explain well the reasons for the insufficient information provided in the question. While taking the test, the subject was not careful enough in reading the question, leading to an error in determining the circle's equation. The subject also struggled to write down their thoughts on the answer sheet. This situation aligns with the characteristics of auditory learners, who struggle with writing but excel at storytelling (DePorter & Hernacki, 2008). Thus, it can be concluded that AS met the first indicator. The following is a description of AS's mathematical reflective

thinking ability in solving mathematical problems for the second indicator, as shown in Figure 7.

Handwritten mathematical work for Figure 7. The work is organized into three columns. The first column shows the elimination of y from the system $y - y_1 = x - x_1$ and $y_2 - y_1 = x_2 - x_1$ to find x . The second column shows the elimination of x to find y . The third column shows the calculation of the discriminant $D = b^2 - 4ac$ for the quadratic equation $x^2 + (-2x + 22)^2 = 100$. The final conclusion states that the object will still be detected by the radar because the discriminant is greater than zero.

English translation: The officer's opinion is incorrect because the discriminant of the line is greater than zero. Therefore, the aircraft's path intersects the radar circle and the object will still be detected by the radar.

Figure 7. Auditory subject response to the 2nd indicator

Figure 7 shows that AS provided the correct answer. The subject identified the mathematical concepts in the given problem by proving that the TNI member's argument was incorrect. Based on the interview results, the subject provided a clear and accurate explanation, identified the errors in the problem statement, and justified the corrections made. This situation is consistent with the findings of Kartono et al. (2019), who reported that auditory learners can write proof correctly and explain the proof process in detail. Thus, it can be said that AS met the second indicator. The following is a description of AS's mathematical reflective thinking ability in solving mathematical problems for the third indicator, as shown in Figure 8.

Handwritten mathematical work for Figure 8. The work shows the calculation of the discriminant $D = b^2 - 4ac$ for the quadratic equation $x^2 + y^2 = r^2$. The resulting values for x and y are calculated, and the final conclusion states that the object is located exactly on the boundary of the yellow zone.

English translation: Trial 1: in the yellow zone. Trial 2: outside the yellow zone. Trial 3: exactly on the boundary of the yellow zone. Arrows 1 and 2 enter yellow zone A. The information is sufficient.

Figure 8. Auditory subject response to the 3rd indicator

Figure 8 shows that AS can identify relevant and irrelevant data and determine the conditions required to solve the given problem. However, on the provided answer sheet,

the subject changed the order of the experiments, resulting in a conclusion that differed from what it should have been. Based on the interview results, AS was able to clearly explain each point they wrote down. Consistent with the findings of Pamungkas et al. (2018), students with an auditory learning style can explain the information in the questions in detail. This situation also aligns with the characteristics of individuals with an auditory learning style, who enjoy speaking, discussing, and explaining things (De-Porter & Hernacki, 2008). Thus, it can be concluded that AS has met the third indicator. The following is a description of AS's mathematical reflective thinking ability in solving mathematical problems for the fourth indicator, as shown in Figure 9.

Handwritten mathematical work for Figure 9:

4a. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\sqrt{(25 - 15)^2 + (-17 - 13)^2}$
 $\sqrt{10^2 + (-30)^2}$
 $\sqrt{100 + 900}$
 $\sqrt{1000} = 31.62$

4b. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\sqrt{(25 - 15)^2 + (15 - 15)^2}$
 $\sqrt{10^2 + 0^2}$
 $\sqrt{100} = 10$

4c. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\sqrt{(25 - 15)^2 + (15 - 15)^2}$
 $\sqrt{10^2 + 0^2}$
 $\sqrt{100} = 10$

4d. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\sqrt{(25 - 15)^2 + (15 - 15)^2}$
 $\sqrt{10^2 + 0^2}$
 $\sqrt{100} = 10$

4e. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $\sqrt{(25 - 15)^2 + (15 - 15)^2}$
 $\sqrt{10^2 + 0^2}$
 $\sqrt{100} = 10$

English translation: Point of tangency. The two circles do not intersect. The average of the two coordinates and the coordinates of the two aircraft wheels are calculated. Radius: 20 cm; diameter: 40 cm.

Figure 9. Auditory subject response to the 4th indicator

Figure 9 shows that AS can draw analogies with reasons and explanations. AS can identify the given case and conclude it. The subject can also provide examples similar to the given case. Based on the interview results, AS is already able to provide examples similar to the case in question. Thus, it can be said that AS meets the fourth indicator. This situation aligns with the results of a study by Haryati et al. (2017), which found that students with an auditory learning style can draw analogies from two similar cases. The following is a description of AS's mathematical reflective thinking ability in solving mathematical problems for the fifth indicator, as shown in Figure 10.

Handwritten mathematical work for Figure 10:

5. $x^2 + y^2 = r^2$

Figure 10. Auditory subject response to the 5th indicator

Figure 10 shows that AS was unable to generalize and provide reasons or explanations, as evidenced by his inability to complete the given table. Based on the interview results, AS did not demonstrate the ability to generalize within the available testing time. The

subject also stated that, in his view, the equation of the tangent line he knew was merely memorized. This situation aligns with the findings of Pamungkas et al. (2018), who found that students with an auditory learning style were unable to provide reasons or explanations for their generalizations. Thus, it can be concluded that AS has not yet met the fifth indicator.

Subject with a Kinesthetic Learning Style

The kinesthetic participant (KS) demonstrates his learning style by enjoying the process of working through problems and not giving up easily when solving them. KS also enjoys it when the math problems given in class relate to everyday life; he says that if he can experience the problem firsthand or visualize the situation, it helps him better understand the material. This finding supports the view that kinesthetic learners prefer learning situations involving movement and physical activity (Sanggorani et al., 2024).

The following are the results of the mathematical reflective thinking ability test for students with a kinesthetic learning style in solving problems on the first indicator shown in Figure 11.

1. b. lingkaran Besar :

jari' lingkaran = 7cm karena setengah dari sisi persegi

keliling lingkaran = $2\pi r = 2 \cdot 3,14 \cdot 7\text{cm} = 43,96\text{ cm}$

luas daerah lingkaran = $\pi r^2 = \frac{22}{7} \cdot (7\text{cm})^2 = 154\text{ cm}$

Persamaan lingkaran = $x^2 + y^2 = r^2 \rightarrow x^2 + y^2 = 7^2$
 $x^2 + y^2 = 49\text{ cm}$

Kurang informasi untuk mencari r lingkaran karena kurang simetris

English translation: Large circle: the radius is 7 cm because it is half the side of the square. The circumference, area, and circle equation are calculated. There is insufficient information to determine the small circle because its symmetry is incomplete.

Figure 11. Kinesthetic subject response to the 1st indicator

Figure 11 shows that KS provided the correct answer. However, the subject's reasoning did not clearly address the mathematical concepts involved. Based on the interview results, KS first identified the known elements and the question posed in the problem. This situation aligns with the findings of Pamungkas et al. (2018), who stated that students with a kinesthetic learning style can identify the mathematical concepts involved in non-trivial math problems. Additionally, the subject tended to answer the researcher's questions briefly, so the researcher needed to ask questions in a step-by-step manner. This situation aligns with the characteristics of kinesthetic learners, who do not write down information in full but can explain it during the interview (Kartono et al., 2019). Thus, it can be concluded that KS met the first indicator. The following

is a description of KS's mathematical reflective thinking ability in solving mathematical problems for the second indicator, as shown in Figure 12.

2. pernyataan TNI benar karena radar hanya mendeteksi benda sejauh 10 km dari markas. Sedangkan benda asing berada lebih dari 10 km namun disebut salah juga karena...

$$\begin{aligned} y - y_1 &= x - x_1 & x^2 + y^2 &= 10^2 & 6.9 < 10 & \text{berarti radar mendeteksi} \\ y_1 - y_2 &= x_1 - x_2 & x^2 + (-2x + 22)^2 &= 100 & & \text{maka garis bersinggungan} \\ y - 22 &= x - 0 & x^2 + 4x^2 - 88x + 484 &= 100 & & \text{dengan lingkaran} \\ 0 - 22 &= 11 - 0 & 5x^2 - 88x + 384 &= 0 & & \text{jadi benda asing terdeteksi radar} \\ y - 22 &= x & & & & \\ -22 &= 11 & D = \frac{B^2 - 4AC}{2 \sqrt{B^2 - 4AC}} & & & \\ -22x &= 11y - 242 & = \frac{88^2 - 4 \cdot 5 \cdot 384}{2 \sqrt{88^2 - 4 \cdot 5 \cdot 384}} & & & \\ = 11y &= 22x - 242 & = \frac{7744 - 7680}{2 \sqrt{7744 - 7680}} & & & \\ y &= 2x + 22 & = 6.9 & & & \end{aligned}$$

English translation: The TNI statement is correct because the radar can only detect objects up to 10 km away, whereas the foreign object is more than 10 km away. However, it can also be considered incorrect because the line intersects the circle; therefore, the foreign object is detected by the radar.

Figure 12. Kinesthetic subject response to the 2nd indicator

Figure 12 shows that KS wrote two conclusions for the answer. The subject wrote that the statement made by the TNI member could be considered both true and false. The subject provided reasons for both of the answers he gave. In line with [Suharna's \(2018\)](#) view, a characteristic of a reflective thinker is that they are always uncertain about the answers they have obtained and seek further answers to find the most accurate one. Based on the interview results, the subject appeared quite uncertain in understanding the given problem. Nevertheless, the subject was able to provide a good explanation for both answers and could correct the errors. This situation aligns with the findings of [Jaenudin et al. \(2017\)](#), who stated that students with a kinesthetic learning style are capable of evaluating the validity of an argument. Thus, it can be said that KS met the second indicator. The following is a description of KS's mathematical reflective thinking ability in solving mathematical problems for the third indicator, as shown in Figure 13.

3. percobaan 1 = $x^2 + y^2 = r^2$ percobaan 2 = $x^2 + y^2 = r^2$
 $\downarrow$ $(-12)^2 + (5)^2 = 17^2$ $17^2 + (-9)^2 = 17^2$
 didalam O kuning $144 + 25 = 169$ 289 $81 > 0$
 $169 < 289$ diluar O kuning
 $-120 < 0$
 $<$
 percobaan 3 = $x^2 + y^2 = r^2$
 $6^2 + 15^2 = 17^2$ didalam O kuning
 $36 + 225 = 261$ karena bersinggungan
 $0 = 0$
 jawabannya A.

English translation: Trial 1: inside the yellow circle. Trial 2: outside the yellow circle. Trial 3: inside the yellow circle because it is tangent. The answer is A.

Figure 13. Kinesthetic subject response to the 3rd indicator

Figure 13 shows that KS identified relevant data and determined the conditions required to solve the given problem. However, on the answer sheet provided, KS did not write down the conclusion. Based on the interview results, KS forgot to write the conclusion on the answer sheet but still explained it well. This situation aligns with the research findings of Nabilah et al. (2022), who found that kinesthetic learners can state conclusions but do not write them down when answering questions. Thus, it can be said that KS has met the third indicator. The following is a description of KS's mathematical reflective thinking ability in solving mathematical problems for the fourth indicator, as shown in Figure 14.

Handwritten work for Figure 14:

$$\begin{aligned} & 2. 4. 1. (x+15)^2 + (y-13)^2 = 29^2 \\ & (x+15)^2 + (y-13)^2 = 576 \\ & 2. (x-25)^2 + (y+17)^2 = 576 \end{aligned}$$

Vertical calculations on the right side:

$$\begin{array}{r} 29 \\ 29 \times \\ \hline 96 \\ 96 + \\ \hline 516 \end{array}$$

Conclusion: *Benda yg sama adalah Motor*

English translation: The same object is the motorcycle.

Figure 14. Kinesthetic subject response to the 4th indicator

Figure 14 shows that KS is not yet able to draw analogies accompanied by reasoning and explanations. The subject is not yet able to conclude the given cases, which prevents the subject from making analogies. Based on the interview results, KS does not know the correct procedure for solving the problem. The example provided by the subject is merely a guess, not based on the mathematical concepts involved in the given problem. This situation aligns with Nurdalilah (2021), research findings, which showed that students with a kinesthetic learning style can perform calculations but cannot provide similar analogies. Thus, it can be said that SK has not yet met the fourth indicator. The following is a description of SK's mathematical reflective thinking ability in solving mathematical problems for the fifth indicator, as shown in Figure 15.

Handwritten work for Figure 15:

$5. 2. x^2 + y^2 = 25$	$(3, -4)$	$3x + (-4)y = 25$
$x^2 + y^2 = 20$	$(4, 2)$	$2x + y = 10$
$x^2 + y^2 = 160$	$(8, 2)$	$24x + y = 50$
$x^2 + y^2 = 5^2$	(x_1, y_1)	$x_1 \cdot x + y_1 \cdot y = 5^2$

Conclusion: *h. kesimpulannya semua tabel yang kosong bisa diisi*

English translation: Conclusion: all empty cells in the table can be filled in.

Figure 15. Kinesthetic subject response to the 5th indicator

Figure 15 shows that KS is not yet able to generalize and provide reasons or explanations. The subject completed the given table, but the solution process was not evident on the answer sheet. Based on the interview results, SK's knowledge is limited to rote memorization. It prevents the subject from generalizing. This situation aligns with the findings of Pamungkas et al. (2018), who stated that students with a kinesthetic learning style cannot provide explanations for their answers when generalizing a case. Thus, it can be concluded that SK has not yet met the fifth indicator.

Overall, the findings show differences in mathematical reflective thinking across the three participants. However, these differences may also be influenced by factors other than learning styles, such as prior mathematical ability, item difficulty, time constraints, and written communication skills. Since this study involved only three participants, the findings should be interpreted as evidence from individual cases rather than generalized to all students with visual, auditory, or kinesthetic learning styles. Future research involving more participants and diverse educational contexts is needed to provide broader insights into students' mathematical reflective thinking.

CONCLUSION AND RECOMMENDATION

Based on the findings, the three participants demonstrated different levels of mathematical reflective thinking. The auditory participant fulfilled four indicators, the kinesthetic participant fulfilled three, and the visual participant fulfilled two. All participants achieved the first and third indicators but failed to meet the fifth indicator, indicating that their reflective thinking was generally limited to understanding and analysis rather than generalization. These findings contribute to mathematics education by providing insights into students' reflective mathematical thinking across different learning styles and may support teachers in designing more adaptive learning activities to foster reflective thinking and reasoning skills.

Recommendations: Teachers need to provide instruction that emphasizes generalization so that students are not only able to understand and analyze, but also to draw deeper, more general conclusions. Future researchers are also encouraged to examine mathematical reflective thinking ability in other learning materials or approaches tailored to students' learning styles.

Artificial Intelligence Statement: The authors declare that no generative artificial intelligence or AI-assisted technologies were used in conducting the research or preparing the submitted manuscript.

Author Contribution Statement: FDADU: Conceptualization, Methodology, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Project administration. S: Validation, Supervision.

Funding Statement: This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Conflict of Interest Statement: The authors declare no conflict of interest.

REFERENCES

- Amaliyah, S., Munawwir, Z., & Puspitasari, Y. (2021). Analisis kemampuan berpikir reflektif dan disposisi matematis siswa ditinjau dari gaya belajar. *Jurnal IKA PGSD (Ikatan Alumni PGSD) UNARS*, 10(2), 305–321. <https://doi.org/10.36841/pgsdunars.v10i2.1429>
- Balta, E. E. (2018). Reflective thinking tendencies and epistemological beliefs in terms of learning styles. *International Journal of Higher Education*, 7(6), 106–117. <https://doi.org/10.5430/ijhe.v7n6p106>
- Chairunnisa, S. (2021). Analisis kemampuan berpikir reflektif matematis siswa SMA menyelesaikan soal program linear ditinjau dari disposisi matematis kriteria Polking. *Prosiding Seminar Nasional Integrasi Matematika dan Nilai Islami*, 4(1), 66–77. <https://digilib.uinsgd.ac.id/43448/>
- Costa, R. D., Souza, G. F., Valentim, R. A. M., & Castro, T. B. (2020). The theory of learning styles applied to distance learning. *Cognitive Systems Research*, 64, 134–145. <https://doi.org/10.1016/j.cogsys.2020.08.004>
- Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). SAGE Publications. <https://books.google.com/books?id=btwENORfhgC>
- DePorter, B., & Hernacki, M. (2008). *Quantum learning: Membiasakan belajar nyaman dan menyenangkan*. Kaifa. https://perpus.smpn7mlg.sch.id/index.php?id=8294&p=show_detail
- Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educative process*. D.C. Heath. <https://archive.org/details/dli.ernet.240488>
- Fuady, A. (2017). Berfikir reflektif dalam pembelajaran matematika. *Jurnal Ilmiah Pendidikan Matematika*, 1(2), 104–112. <https://doi.org/10.26877/jipmat.v1i2.1236>

- Halean, S., Kandowangko, N., & Goni, S. Y. V. I. (2021). Peranan pendidikan dalam meningkatkan sumber daya manusia di SMA Negeri 1 Tampan Amma di Talaud. *Jurnal Holistik*, 14(2), 1–15. <https://ejournal.unsrat.ac.id/v3/index.php/holistik/article/view/33774>
- Haryati, T., Nindiasari, H., & Sudiana, R. (2017). Analisis kemampuan dan disposisi berpikir reflektif matematis siswa ditinjau dari gaya belajar. *JPPM*, 10(2), 146–158. <https://doi.org/10.30870/jppm.v10i2.2039>
- Hendriana, H., Putra, H. D., & Hidayat, W. (2019). How to design teaching materials to improve the ability of mathematical reflective thinking of senior high school students in Indonesia? *Eurasia Journal of Mathematics, Science and Technology Education*, 15(12). <https://doi.org/10.29333/ejmste/112033>
- Jaenudin, J., Nindiasari, H., & Pamungkas, A. S. (2017). Analisis kemampuan berpikir reflektif matematis siswa ditinjau dari gaya belajar. *Prima: Jurnal Pendidikan Matematika*, 1(1), 69–82. <https://doi.org/10.31000/prima.v1i1.256>
- Kartono, A., Mariani, P. D., & Mariani, S. (2019). Analysis of students' mathematical reflective thinking on problem based learning (PBL) based from learning styles. *Unnes Journal of Mathematics Education*, 8(1), 34–41. <https://doi.org/10.15294/ujme.v8i1.24239>
- King, F., Goodson, L., & Rohani, F. (1997). *Higher order thinking skills*. Educational Services Program. <https://dl.icdst.org/pdfs/files1/edb06d7f36371f4f909aab603b2c51c0.pdf>
- Maulida, N. S. S., Happy, N., & Sugiyono, E. (2024). Analisis kemampuan berpikir reflektif dalam menyelesaikan masalah. *Didaktik: Jurnal Ilmiah PGSD FKIP Universitas Mandiri*, 10(2), 677–687. <https://doi.org/10.36989/didaktik.v10i2.2854>
- Muntazhimah, M., Turmudi, T., & Prabawanto, S. (2021). The relation between prior knowledge and students' mathematics reflective thinking ability. *Journal of Physics: Conference Series*, 1731, Article 012043. <https://doi.org/10.1088/1742-6596/1731/1/012043>
- Nabilah, Amrullah, Lu'luilmaknun, U., & Sripatmi. (2022). Analisis kemampuan berpikir reflektif matematis siswa ditinjau dari gaya belajar. *Journal of Classroom Action Research*, 5(1), 185–191. <https://doi.org/10.29303/jcar.v5i1.2643>
- Nurdalilah. (2021). Analisis kemampuan berpikir reflektif matematika dengan gaya belajar visual, audiotorial dan kinestetik siswa. *MathEdu (Mathematics Education Journal)*, 4(3), 64–71. <https://doi.org/10.37081/mathedu.v4i3.3143>
- Pamungkas, A. S., Mentari, N., & Nindiasari, H. (2018). Analisis kemampuan berpikir reflektif siswa SMP berdasarkan gaya belajar. *NUMERICAL: Jurnal Matematika dan Pendidikan Matematika*, 2(1), 69. <https://doi.org/10.25217/numerical.v2i1.209>

- Ramadhani, R., & Juandi, D. (2020). An analysis of mathematical reflective thinking skills of senior high school students. *Journal of Physics: Conference Series*, 1521, Article 032059. <https://doi.org/10.1088/1742-6596/1521/3/032059>
- Ratnaningsih, N., & Hidayat, E. (2020). Reflective mathematical thinking process and student errors: An analysis in learning style. *Journal of Physics: Conference Series*, 1613, Article 012037. <https://doi.org/10.1088/1742-6596/1613/1/012037>
- Salido, A., & Dasari, D. (2019). The analysis of students' reflective thinking ability viewed by students' mathematical ability at senior high school. *Journal of Physics: Conference Series*, 1157, Article 022121. <https://doi.org/10.1088/1742-6596/1157/2/022121>
- Sanggorani, A., Hendrowati, T. Y., & Rahayu, S. (2024). Kemampuan berfikir reflektif matematis siswa ditinjau dari gaya belajar. *Jurnal e-DuMath*, 10(1), 11–16. <https://doi.org/10.52657/je.v10i1.2240>
- Sholikhin, R., Nur Afifah, D. S., & Maryono, M. (2021). Students' reflective thinking in mathematical problem solving. *MaPan*, 9(1), 153–166. <https://doi.org/10.24252/mapan.2021v9n1a10>
- Suharna, H. (2018). *Teori berpikir reflektif dalam menyelesaikan masalah matematika*. Deepublish. <https://deepublishstore.com/produk/buku-teori-berpikir/>
- Sumarmo, U. (2016). *Pedoman pemberian skor pada beragam tes kemampuan matematik*. STKIP Siliwangi. <https://id.scribd.com/document/361074178/Pedoman-Pemberian-Skor-Pada-Beragam-Tes-Kemampuan-Matematik>
- Thahir, A., Komarudin, Hasanah, U. N., & Rahmahwaty. (2019). MURDER learning models and self-efficacy: Impact on mathematical reflective thinking ability. *Journal for the Education of Gifted Young Scientists*, 7(4), 1120–1133. <https://doi.org/10.17478/jegys.594709>
- Widodo, S. A., Wijayanti, A., Irfan, M., Pusporini, W., Mariah, S., & Rochmiyati, S. (2023). Effects of worksheets on problem-solving skills: Meta-analytic studies. *International Journal of Educational Methodology*, 9(1), 151–167. <https://doi.org/10.12973/ijem.9.1.151>
- Zhang, J., & Dempsey, P. (2019). Exploration and confirmation of a reflective-thinking scale to measure transformative learning in online courses. *Assessment and Evaluation in Higher Education*, 44(3), 463–475. <https://doi.org/10.1080/02602938.2018.1520194>