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The Effect of Guided Discovery Learning on Students' Analytical Thinking Skills in Relations and Functions

Yanuar Sofyan Erlangga[1], *Agung Putra Wijaya[2], Nurain Suryadinata[3]
[1–3] University of Lampung, Bandar Lampung, Indonesia
agung.wijaya@fkip.unila.ac.id

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ABSTRACT Analytical thinking skills are essential for students to dissect complex mathematical structures, yet teacher-centered learning often fails to foster these higher-order cognitive processes. This study examined the effect of the Guided Discovery Learning (GDL) model on eighth-grade students' analytical thinking skills in Relations and Functions. Using a quasi-experimental non-equivalent control-group design, two intact eighth-grade classes at SMP Negeri 19 Bandar Lampung were selected through cluster random sampling: class VIII.E ($n_1 = 32$) as the experimental group receiving GDL instruction and class VIII.F ($n_2 = 32$) as the control group receiving direct instruction. Data were collected using a pretest and posttest consisting of four validated essay prompts aligned with Anderson and Krathwohl's revised Bloom's taxonomy indicators. Data were analyzed using descriptive statistics and an independent-samples t-test on N-gain scores. The results revealed that students instructed with GDL achieved a significantly higher mean posttest score ($M = 68.44$, $SD = 8.12$) and a higher mean N-gain ($M_{\text{N-gain}} = 0.37$, $SD = 0.12$, medium category) than students in the control group ($M = 61.25$, $SD = 7.45$; $M_{\text{N-gain}} = 0.26$, $SD = 0.11$, low category). The independent-samples t-test confirmed a significant difference in N-gain between the two groups ($t(62) = 3.824$, $p < 0.001$, two-tailed). Furthermore, the intervention yielded a large effect size (Cohen's $d = 0.96$). Analysis of specific indicators showed that the greatest improvement occurred in the differentiating indicator, followed by organizing and attributing. These findings suggest that GDL effectively enhances students' analytical thinking skills by providing structured scaffolding during mathematical exploration.

Keywords: analytical thinking skills, guided discovery learning, normalized gain, relations and functions

INTRODUCTION

Analytical thinking skills are a critical component of higher-order thinking in mathematics education. According to the revised Bloom's taxonomy proposed by Anderson and Krathwohl (2001), analytical thinking involves breaking down material into its constituent parts, determining how these parts relate to one another and to an overall structure, and identifying underlying motives or perspectives. In secondary school mathematics, these skills enable students to differentiate between relevant and irrelevant information, organize mathematical data into structured representations, and attribute underlying mathematical properties or functions to observed patterns (Cahyani et al., 2024; Putri & Syarifuddin, 2018; Satiti & Wulandari, 2021; Mahyastuti et al., 2021). Without well-developed analytical skills, students struggle to progress from procedural fluency to conceptual understanding and non-routine problem solving.

Despite the recognized importance of analytical thinking, empirical evidence indicates that secondary students in Indonesia continue to face persistent difficulties in higher-order mathematical reasoning. International benchmark studies, such as the Programme for International Student Assessment (PISA) 2022, reported that Indonesian 15-year-old students achieved a mean mathematics score of 366, substantially below the OECD average of 472 (OECD, 2023). While PISA measures general mathematical literacy across varied contexts rather than analytical thinking exclusively, successfully solving PISA items, particularly those at Level 3 and above, inherently requires analytical processes such as dissecting word problems, organizing data tables, and identifying functional relationships (OECD, 2023; Machromah et al., 2020).

To investigate these difficulties at the local institutional level, a preliminary diagnostic study was conducted on August 12, 2025, involving 32 eighth-grade students in class VII.H at SMP Negeri 19 Bandar Lampung. Students were administered a diagnostic test comprising three essay problems on foundational algebra and functional reasoning, structured around Anderson and Krathwohl's (2001) analytical indicators. The diagnostic results revealed that only 5 out of 32 students (15.63%) achieved the minimum mastery threshold for analytical thinking. Specifically, 81.25% of students failed to differentiate between domain, codomain, and range in word problems, and 75.00% struggled to organize given ordered pairs into algebraic formulas. Classroom observations during the diagnostic phase indicated that instruction was predominantly teacher-centered, focusing heavily on rote formula memorization and algorithmic drills. Consequently, students exhibited cognitive passivity when confronted with non-routine functional tasks.

To address these pedagogical shortcomings, an instructional approach that actively engages students in knowledge construction is necessary. The Guided Discovery Learning (GDL) model, grounded in Bruner's constructivist learning theory, offers a promising alternative (Bruner, 1961). GDL engages learners in problem-solving activities in which they explore mathematical phenomena, formulate hypotheses, analyze empirical patterns, and derive general rules under the structured guidance and scaffolding of the teacher (Alfieri et al., 2011; Veermans et al., 2006). The GDL learning sequence typically comprises six operational phases: (1) stimulation, (2) problem statement, (3) data collection, (4) data processing, (5) verification, and (6) generalization (Syah, 2020; Siregar et al., 2020).

Previous empirical studies have documented the positive effects of discovery-based instruction across various educational contexts. For instance, Simamora et al. (2019) demonstrated that guided discovery improved students' mathematical problem-solving ability in middle school. Similarly, Khairunnisa & Juandi (2022) found that discovery learning enhanced students' critical thinking skills. However, a significant research gap remains regarding the targeted effect of GDL on specific analytical thinking indicators—namely, differentiating, organizing, and attributing—when students learn abstract algebraic concepts such as Relations and Functions in junior high school. The topic of Relations and Functions serves as a vital bridge between arithmetic and abstract functional analysis, making it particularly suited for GDL. Students must differentiate mapping properties, organize input-output relationships across multiple representations (diagrams, tables, sets, and algebraic equations), and attribute structural rule patterns to mathematical functions.

Therefore, the objective of this study was to examine the effect of the GDL model on eighth-grade students' analytical thinking skills in the topic of Relations and Functions. Specifically, this study aimed to determine whether students instructed using the GDL model achieved significantly higher analytical thinking gain scores than those receiving conventional direct instruction and to analyze student performance across specific analytical thinking indicators.

RESEARCH METHOD

Research Design

This study employed a quantitative approach with a quasi-experimental non-equivalent control group design (Creswell & Creswell, 2018). This design was selected because complete random assignment of individual students to new classes was not feasible due to administrative constraints and the need to preserve intact classroom environments at the participating school. The structural notation of the research design is presented in Figure 1.

Group A (Experimental)	O_1	X	O_2
Group B (Control)	O_3	-	O_4

Figure 1. Non-equivalent Control Group Design
(O_1, O_3: Pretests; X: Guided Discovery Learning intervention;
- : Conventional Direct Instruction; O_2, O_4: Posttests)

Population and Sample

The accessible population comprised all eighth-grade students enrolled at SMP Negeri 19 Bandar Lampung during the 2025/2026 academic year, distributed across eight regular classes (VIII.C to VIII.J), totaling 251 students. Classes VIII.A and VIII.B were purposively excluded prior to sampling because they were designated as academic honors/accelerated classes and thus did not reflect the baseline heterogeneity of the regular student population. Cluster random sampling was conducted using a simple lottery method. Class VIII.E ($n_1 = 32$) was randomly selected as the experimental group, receiving GDL instruction, and class VIII.F ($n_2 = 32$) was selected as the control group, receiving conventional direct instruction. The total sample size across both groups was $N = 64$.

Ethical Considerations

Institutional approval was obtained from the administration of SMP Negeri 19 Bandar Lampung. Prior to data collection, informed consent forms were distributed to parents/guardians, and assent was obtained from all participating students. Student identities and assessment scores were kept strictly confidential through anonymous coding.

Instructional Interventions

Both groups were taught by the same mathematics teacher over four instructional sessions (2×40 minutes per session) focusing on the topic of Relations and Functions.

Experimental Group (GDL)

Instruction followed the six phases of Guided Discovery Learning (Syah, 2020). Students worked in small heterogeneous groups (4–5 students) using student worksheets specifically designed to scaffold exploration. Students investigated real-world scenarios, collected tabular input-output data, processed the data to identify functional rules, verified their conjectures with peer groups, and presented generalized mathematical formulas to the class. The teacher acted as a facilitator, providing targeted prompts (scaffolding) rather than direct answers.

Control Group (Conventional Direct Instruction)

Instruction followed a traditional teacher-centered approach. The teacher presented definitions, rules, and algebraic formulas directly on the whiteboard, demonstrated step-by-step example solutions, assigned individual practice problems from the standard textbook, and reviewed solutions at the front of the class.

Research Instrument and Validation

Data were collected using an analytical thinking skills test administered as a pretest before the intervention and a posttest following the four-session intervention. The instrument consisted of four open-ended essay items designed to evaluate analytical thinking skills based on the framework adapted from Anderson and Krathwohl (2001). The alignment of the test items with the analytical indicators and mathematical subtopics is presented in Table 1.

Table 1. Specification Grid of the Analytical Thinking Skills Test

Item No	Sub-Topic	Target Analytical Indicator	Behavioral Description	Max Score
1	Concept of Relations & Mappings	Differentiating	Distinguish between relations that are functions and non-functions from sets of ordered pairs and arrow diagrams.	25
2	Representation of Functions	Organizing	Convert relation data from verbal descriptions into arrow diagrams, Cartesian graphs, and structured tables.	25
3	Function Formulas ($f(x) = ax + b$)	Attributing	Determine the linear functional rule/formula by attributing underlying a and b coefficients from given input-output pairs.	25
4	Problem Solving in Functions	Differentiating & Organizing	Extract relevant problem parameters from a real-world scenario and organize an algebraic model to solve non-routine tasks.	25

The test instrument underwent rigorous content and construct validation by two university mathematics education experts and one senior secondary school mathematics teacher. Content validity was evaluated using Aiken's V formula, yielding an overall index of $V = 0.88$, which indicated strong content validity. The instrument was subsequently field-tested with a trial group of 30 ninth-grade students who had previously completed the topic. Construct validity for individual test items was confirmed using Pearson product-moment correlation ($r_{\text{count}} > r_{\text{table}} = 0.361$, $\alpha = 0.05$). Empirical reliability was evaluated for the overall test using Cronbach's alpha, yielding $\alpha = 0.785$, which exceeded the standard reliability threshold of 0.70.

Data Analysis

To measure individual skill growth while accounting for baseline variance in pretest performance, the normalized gain (N-gain) score was calculated for each student using Hake's (1998) formula:

$$\text{N-gain} = \frac{S_{\text{post}} - S_{\text{pre}}}{S_{\text{max}} - S_{\text{pre}}}$$

where S_{post} is the posttest score, S_{pre} is the pretest score, and S_{max} is the maximum possible score (100). The average N-gain was categorized according to Hake's criteria: High (N-gain ≥ 0.70), Medium ($0.30 \leq$ N-gain < 0.70), and Low (N-gain < 0.30).

To examine student growth across specific analytical thinking indicators (differentiating, organizing, and attributing), indicator gains were calculated by subtracting the pretest mean percentage score from the posttest mean percentage score (Δ = Posttest % − Pretest %). Indicator gains were categorized based on percentage-point improvement criteria adapted from Hake (1998) and Putri & Syarifuddin (2018): High ($\Delta \geq 20.00$%), Moderate ($10.00\% \leq \Delta < 20.00$%), and Low ($\Delta < 10.00$%).

Statistical hypotheses were framed as follows:

H_0: $\mu_1 \leq \mu_2$ (The mean N-gain in analytical thinking skills for students in the GDL group is less than or equal to that of students in the conventional instruction group).

H_1: $\mu_1 > \mu_2$ (The mean N-gain in analytical thinking skills for students in the GDL group is significantly higher than that of students in the conventional instruction group).

Tests of parametric assumptions were conducted prior to hypothesis testing. The Shapiro-Wilk test was conducted for normality, and Levene's test was used to assess homogeneity of variance. An independent-samples t-test ($\alpha = 0.05$, directional) was conducted on the N-gain data using SPSS version 26.0. Additionally, the magnitude of the instructional treatment effect was quantified using Cohen's d:

$$\text{Cohen's } d = \frac{M_1 - M_2}{SD_{\text{pooled}}}$$

where

$$SD_{\text{pooled}} = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}}.$$

Cohen's d effect sizes were interpreted as small (0.20), medium (0.50), or large (≥ 0.80) (Cohen, 1988).

RESULTS AND DISCUSSION

Pretest and Posttest Analytical Thinking Performance

Descriptive statistics summarizing the pretest, posttest, and N-gain scores for the experimental ($n_1 = 32$) and control ($n_2 = 32$) groups are presented in Table 2.

Table 2. Descriptive Statistics of Assessment Scores across Groups

Class	Assessment	Sample (n)	Min Score	Max Score	Ideal Max	Mean (M)	Std. Deviation (SD)
Experimental	Pretest	32	30.00	65.00	100.00	48.75	8.42
	Posttest	32	55.00	85.00	100.0	68.44	8.12
	N-gain	32	0.14	0.62	1.00	0.37	0.12
Control	Pretest	32	30.00	62.00	100.00	47.19	7.89
	Posttest	32	48.00	78.00	100.00	61.25	7.45
	N-gain	32	0.05	0.48	1.00	0.26	0.11

As indicated in Table 2, both groups exhibited comparable baseline analytical thinking skills on the pretest ($M_{\text{exp}} = 48.75$, $SD = 8.42$; $M_{\text{con}} = 47.19$, $SD = 7.89$). Following instruction, the experimental group achieved a posttest mean score of 68.44 ($SD = 8.12$), whereas the control group achieved a posttest mean score of 61.25 ($SD = 7.45$). The mean N-gain for the GDL group was 0.37 ($SD = 0.12$), falling into the medium improvement category. In contrast, the control group yielded a mean N-gain of 0.26 ($SD = 0.11$), classified in the low category.

Assumptions Testing and Statistical Hypothesis Testing

Parametric statistical assumptions were evaluated prior to conducting the hypothesis test. Results of the Shapiro-Wilk test demonstrated that N-gain distributions for both the experimental group ($W(32) = 0.968$, $p = 0.441$) and the control group ($W(32) = 0.955$, $p = 0.201$) did not significantly deviate from normality ($p > 0.05$). Furthermore, Levene's test confirmed equal variances across groups ($F(1, 62) = 0.184$, $p = 0.669$).

Because the parametric assumptions were satisfied, an independent-samples t-test was conducted on the N-gain scores. The inferential statistical results are presented in Table 3.

Table 3. Independent-Samples t-Test Results for Normalized Gain Scores

Group	Mean	SD	Mean Difference	t-value	df	p-value (2-tailed)	95% CI of Difference	Cohen's d
Experimental	0.37	0.12	0.110	3.824	62	< 0.001	[0.052, 0.168]	0.96
Control	0.26	0.11						

The independent-samples t-test revealed a statistically significant difference in N-gain performance between the experimental and control groups ($t(62) = 3.824$, $p < 0.001$). Because the two-tailed p-value was well below the threshold of $\alpha = 0.05$, the null hypothesis (H_0) was rejected in favor of H_1. This result indicates that students who learned through the GDL model experienced significantly greater improvements in analytical thinking skills than those who received conventional direct instruction.

Moreover, the standardized effect size calculation yielded Cohen's $d = 0.96$. According to Cohen's (1988) benchmark, a value exceeding 0.80 represents a large effect size. This finding indicates that the GDL intervention had a substantial practical impact on students' skill acquisition, placing the average GDL student approximately 0.96 standard deviations above the average control student in analytical growth.

Analysis of Specific Analytical Thinking Indicators

To further elucidate the pedagogical impact of GDL, students' pretest and posttest performance was analyzed across the three specific analytical thinking indicators adapted from Anderson and Krathwohl (2001). Table 4 summarizes the percentage score gains for each indicator across both instructional groups.

Table 4. Performance Breakdown by Analytical Thinking Indicators

Indicator	Group	Pretest Mean (%)	Posttest Mean (%)	Indicator Gain (%)	Category
Differentiating	Experimental	51.25	74.38	+23.13	High
	Control	49.38	64.38	+15.00	Moderate
Organizing	Experimental	46.88	67.50	+20.62	High
	Control	45.63	60.00	+14.37	Moderate
Attributing	Experimental	48.13	63.44	+15.31	Moderate
	Control	46.56	59.38	+12.82	Moderate

Note: Percentage values represent the mean percentage of maximum possible score earned by students for items corresponding to each indicator.

As shown in Table 4, the experimental group outperformed the control group across all three indicators. First, for the differentiating indicator, the experimental group showed the highest improvement (+23.13%). Students effectively distinguished functions from general relations by evaluating domain-codomain mapping rules, such as verifying that every domain element is assigned exactly one range element. Second, for the organizing indicator, the experimental group achieved a 20.62 percentage-point increase. Guided activity sheets enabled students to translate functional data fluently across multiple representations (mapping diagrams, coordinate tables, and graphs on Cartesian planes). Finally, for the attributing indicator, the experimental group demonstrated a 15.31 percentage-point gain. Although this was the most challenging cognitive indicator for both groups, GDL students were better equipped to analyze linear pattern changes to determine algebraic function rules ($f(x) = ax + b$).

Building on these indicator-level results, the findings demonstrate that the GDL model significantly enhances eighth-grade students' analytical thinking skills in mathematics compared with conventional direct instruction ($t(62) = 3.824$, $p < 0.001$, $d = 0.96$). These results align with constructivist learning theories, which posit that active exploration and cognitive restructuring lead to deeper conceptual organization than the passive reception of facts (Bruner, 1961; Piaget, 1970).

The observed superiority of the GDL group can be attributed to the structural alignment between the phases of GDL and the cognitive demands of analytical thinking (Anderson & Krathwohl, 2001; Syah, 2020). During the stimulation and problem statement phases, students were presented with authentic, non-routine scenarios involving relations, such as matching student IDs to uniform sizes or analyzing taxi fare structures. These tasks forced students to exercise the differentiating indicator by filtering relevant input parameters from background noise.

During the data collection and data processing phases, small groups of students constructed data tables and arrow diagrams. These phases directly exercised the organizing indicator. Rather than receiving ready-made formulas, students actively organized pairs of domain and range values, noticing mathematical regularities across representations. As observed in classroom field notes during Session 3, when students attempted to find the formula for a linear function, group discussions required them to attribute the constant rate of change to coefficient a and the baseline offset to the constant term b. This process embodies the attributing indicator, as students identified the underlying functional structure governing the observed empirical data.

Furthermore, the verification and generalization phases provided critical metacognitive checkpoints. By presenting group discoveries to the class and defending their mathematical logic against peer critique, students refined their analytical reasoning and corrected misconceptions. Scaffolding provided by the teacher served as a cognitive bridge (Vygotsky, 1978), preventing frustration without prematurely revealing procedural shortcuts.

Conversely, students in the control group received direct instruction in which definitions and formulas were introduced at the beginning of the lesson. While this approach fostered procedural efficiency in solving standard algorithmic problems, it offered minimal cognitive opportunities for students to organize unformatted data or attribute structural rules independently. Consequently, when confronted with non-routine posttest prompts requiring multi-step structural analysis, control group students frequently exhibited errors such as misapplying formulas or failing to recognize non-function mappings.

Limitations of the Study

Several methodological limitations of this study should be noted when interpreting the results. First, regarding the sample scope, the study was conducted within a single urban public junior high school in Bandar Lampung and excluded academic honors classes. Caution should therefore be exercised when generalizing these findings to diverse rural schools or students in different academic tracks. Second, regarding curricular constraints, due to unexpected school-level scheduling conflicts during Sessions 3 and 4, two subtopics, specifically Representing Functions and Determining Function Formulas, were combined into a single instructional block. This increase in instructional pace may have elevated cognitive load, potentially limiting improvement in the attributing indicator. Finally, regarding the intervention duration, the study was limited to four instructional sessions (eight academic hours). A longer intervention period may be required to establish lasting cognitive habits in higher-order analytical reasoning.

CONCLUSION AND RECOMMENDATION

This study demonstrates that the GDL model has a statistically significant positive effect on eighth-grade students' analytical thinking skills in the topic of Relations and Functions. Students taught with GDL achieved a significantly higher mean normalized gain ($M_{\text{N-gain}} = 0.37$, medium category) than those taught through conventional direct instruction ($M_{\text{N-gain}} = 0.26$, low category; $p < 0.001$), with a large intervention effect size (Cohen's $d = 0.96$). Detailed indicator analyses demonstrate that GDL effectively fosters students' skills in differentiating mathematical properties, organizing multi-representational data, and attributing underlying functional structures.

Based on these conclusions, mathematics educators are encouraged to adopt GDL as an instructional strategy for teaching abstract algebraic topics. Teachers should design structured discovery worksheets that guide students from concrete representations to abstract mathematical formulas. Future research should investigate the long-term retention of analytical skills fostered by GDL across larger, geographically diverse samples and explore the integration of dynamic mathematics software, such as GeoGebra, to further enhance student exploration during the discovery phases.

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