JURNAL ABSIS
Jurnal Pendidikan Matematika dan Matematika
Vol. 9 No. 1, April 2026

AN EXPLORATION OF STUDENTS' BELIEFS AND SELF-CONFIDENCE IN CONSTRUCTING MATHEMATICAL PROOFS

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ABSTRACT The ability to construct mathematical proofs is an essential competency in university-level mathematics education. However, previous studies have largely focused on cognitive aspects, while the relationship between affective aspects, particularly beliefs and self-confidence, and the process of constructing mathematical proofs remains underexplored. This study aims to examine students' beliefs and self-confidence in constructing mathematical proofs by considering deductive, abductive, and key processes. This study employed a qualitative approach with a case study design. The participants were 42 second-semester students who had completed a Number Theory course. Data were collected through a mathematical proof test and semi-structured interviews. Three students were purposively selected for in-depth analysis. The data were analyzed using thematic analysis to identify patterns of beliefs and self-confidence during the proof construction process. The results indicate that students' beliefs and self-confidence can be classified into three categories, varying in their involvement in the proof construction process. These findings imply the importance of instructional practices that not only emphasize cognitive aspects but also support the development of students' beliefs and self-confidence through activities such as explanation and reflection in proving.

Keywords: mathematical proof, belief, self-confidence, deductive process, abductive process

ABSTRAK Kemampuan mengonstruksi bukti matematis merupakan salah satu kompetensi penting dalam pembelajaran matematika di perguruan tinggi. Pemahaman terhadap proses konstruksi bukti matematis tidak dapat dicapai hanya melalui kajian aspek kognitif, tetapi juga memerlukan perhatian terhadap aspek afektif mahasiswa. Penelitian ini bertujuan untuk mengkaji aspek afektif berupa keyakinan dan kepercayaan diri mahasiswa dalam mengonstruksi bukti matematis. Penelitian menggunakan pendekatan kualitatif dengan desain studi kasus. Subjek penelitian adalah 42 mahasiswa semester II pada sebuah universitas negeri di Indonesia yang telah menempuh mata kuliah Teori Bilangan. Data dikumpulkan melalui tes tertulis berupa soal pembuktian matematis dan wawancara semi-terstruktur. Untuk pendalaman analisis, dipilih tiga mahasiswa secara purposif yang mewakili variasi proses konstruksi bukti matematis. Analisis data dilakukan secara deskriptif-kualitatif dengan menelaah keyakinan dan kepercayaan diri mahasiswa selama proses mengonstruksi bukti matematis. Hasil penelitian menunjukkan bahwa keyakinan mahasiswa terbagi ke dalam tiga kategori berdasarkan setiap proses konstruksi bukti yang dilakukan. Selain itu, kepercayaan diri mahasiswa juga terbagi ke dalam tiga kategori, mulai dari kepercayaan diri yang sejalan dengan pencapaian jawaban secara cepat meskipun tidak tepat hingga kepercayaan diri yang ditunjukkan melalui kemampuan menjelaskan dan mempertahankan proses konstruksi bukti secara runtut dan meyakinkan. Temuan ini menegaskan pentingnya peran aspek afektif, selain aspek kognitif, dalam memahami perilaku mahasiswa dalam mengonstruksi bukti matematis.

Kata-kata kunci: pembuktian matematis, keyakinan, kepercayaan diri, proses deduktif, proses abduktif

INTRODUCTION

Mathematical proof is one of the fundamental activities in mathematics (CadwalladerOlsker, 2011; Hanna, 2000) and plays an important role in the development of students' reasoning, conceptual understanding, and mathematical ways of thinking. Through proof, students not only verify the truth of a mathematical statement (CadwalladerOlsker, 2011; Hersh, 1993; Tall, 1998), but also learn to connect definitions, properties, and theorems in a logical and systematic manner (Selden & Selden, 2008). Therefore, the ability to construct mathematical proofs is an essential competence that must be mastered by students, particularly at the university level (Weber, 2001).

Various studies show that students frequently experience difficulties in constructing mathematical proofs. These difficulties include an inability to understand the statement to be proven (Selden & Selden, 2008), errors in using definitions and theorems (Azrou, 2013; Doruk & Kaplan, 2015; Noto et al., 2019), and failure to organize logical arguments (Selden et al., 2014). Most research on mathematical proof tends to emphasize cognitive aspects, such as reasoning errors, proof strategies, and the structure of arguments developed by students. This approach has made important contributions to understanding how students think when developing mathematical proofs.

However, students' understanding of the process of constructing mathematical proofs cannot be fully captured by focusing solely on cognitive aspects. The proving process also involves affective aspects (Furinghetti & Morselli, 2009), such as confidence in the steps taken and self-confidence in explaining and defending mathematical arguments. In addition, students' beliefs about proof and about themselves as learners of proof are known to play a role in shaping their mathematical behavior when constructing and evaluating proofs (Stylianou et al., 2015). In this context, differences in students' levels of belief and self-confidence may influence how they make decisions and assess the validity of steps in the proving process.

Several studies have examined the role of affective aspects in mathematical proving by emphasizing the interplay between affective and cognitive factors in the proving process. For example, Furinghetti & Morselli (2009) show that beliefs about oneself and about mathematical activity can influence strategy selection, responses to difficulties, and the overall course of the proving process. Furthermore, Selden & Selden (2013) highlight the importance of self-efficacy and persistence in supporting actions during proof construction, such as exploration, revising arguments, and validation. Meanwhile, Viholainen et al. (2019) investigate students' self-efficacy related to their ability to understand, construct, and evaluate proofs, and show that although students are highly motivated, they often remain uncertain about their proving abilities. However, these studies tend to focus on the general role of affective aspects in relation to performance or outcomes of proving, and only limited attention has been given to how students' beliefs and self-confidence are involved in the process of proof construction itself.

In particular, the relationship between affective aspects and the thinking processes occurring during proving—such as deductive processes, abductive processes, and the key processes that bridge the two—remains underexplored. To address this gap, this study conceptualizes the process of constructing mathematical proofs as involving three main components, namely deductive processes, abductive processes, and key processes (Kusnandi, 2008). Deductive processes refer to deriving arguments logically from given information, while abductive processes involve formulating conjectures or intermediate targets that enable reaching a conclusion. The key process functions to bridge the results of deductive and abductive processes in order to form a coherent line of proof. Within this framework, students' ability to integrate these processes may also be associated with their beliefs and self-confidence, which can influence how they engage in the proving process (Shongwe, 2025). Based on this background, this study aims to examine students' beliefs and self-confidence in constructing mathematical proofs by analyzing their engagement in deductive, abductive, and key processes. This study is expected to contribute to a more comprehensive understanding of the role of affective aspects in mathematical proof and to serve as a basis for developing proof-oriented instruction that emphasizes not only cognitive aspects but also the development of students' affective aspects.

METHODS

This study employed a qualitative approach with a case study design to obtain an in-depth understanding of students' beliefs and self-confidence in constructing mathematical proofs (Crowe et al., 2011). This approach was selected because it enables researchers to examine comprehensively the process of mathematical proof construction along with the accompanying affective aspects. The study consisted of three stages: the preparation stage, the data collection stage, and the data analysis stage.

At the first stage, the researcher conducted a literature review related to mathematical proving and affective aspects, particularly students' beliefs and self-confidence in the process of constructing proofs. This review served as the basis for formulating the indicators used to analyze the affective aspects in this study. These indicators were developed based on the abductive–deductive process framework, taking into account the relationship between affective aspects and students' thinking processes in mathematical proving. Students' beliefs in constructing mathematical proofs were identified through their ability to construct logical arguments, provide justification for each step of the proof, and relate given information to the conclusion to be achieved through deductive, abductive, and key processes. Meanwhile, students' self-confidence was identified through interview data, particularly when students were asked to explain and defend the proof construction they had developed. In this study, four mathematical statement problems in basic number theory were used, requiring students to construct proofs independently. These problems were validated by two mathematics education experts to ensure content appropriateness, level of difficulty, and their potential to elicit variations in proof construction processes and students' affective responses, as presented in Table 1.

Table 1. Problem statement

Prove the following statement using two proof strategies (direct and indirect proof): "Let $n \in \mathbb{Z}$. If 3n + 1 is odd, then 2n + 8 is divisible by 4."
Let a, b ∈ $\mathbb{Z}$. Prove that 3 | (5a + 8b) if and only if 3 | (a + b).
Prove that the real number $\sqrt{5}$ is irrational.
Let n be an odd integer and $3 \nmid n$. Show that $24 \mid (n^2 - 1)$.

The second stage was the data collection stage. Data were collected through an individual written test administered for 60 minutes and completed by 42 second-semester students from one class at a public university in Bandung, Indonesia. All students had completed a Number Theory course and were therefore assumed to possess sufficient prior mathematical knowledge to construct mathematical proofs. In this test, students were asked to write a complete proof construction process for each given mathematical statement on the provided answer sheets. During the test, students were not allowed to use smartphones, calculators, or any other electronic devices, so that the proof construction process reflected each student's individual ability.

To obtain a deeper understanding of students' affective aspects, semi-structured interviews were also conducted at this stage. Based on the results of the written test, three students were purposively selected for in-depth analysis and interviews, representing variations in their proof construction processes: a student who was able to perform all three proof construction processes (deductive, abductive, and key process), a student who was able to perform two processes (deductive and abductive), and a student who was able to perform only one process (either

https://doi.org/10.30606/absis.v9i1.4230

deductive or abductive). The interviews were conducted after the written test and focused on exploring students' beliefs regarding the proof steps they used, as well as their self-confidence when explaining and defending their proof construction process.

In this study, students' beliefs in constructing mathematical proofs were identified through their ability to provide arguments and justifications in each stage of the construction process. Meanwhile, students' self-confidence was identified through their verbal explanations during the interviews, as reflected in the fluency of their explanations, their willingness to defend their arguments, and their responses to questions or doubts raised about the proofs they proposed.

The third stage was data analysis. The analyzed data consisted of students' written responses and interview transcripts. The analysis was conducted using a descriptive qualitative approach by examining in depth the characteristics of students' beliefs and self-confidence during the process of constructing mathematical proofs. To enhance the validity of the findings, source and method triangulation were applied by comparing written data and interview data. The results of the analysis were then synthesized to provide a comprehensive description of variations in students' beliefs and self-confidence in constructing mathematical proofs.

FINDING AND DISCUSSION

This section presents and discusses the research findings regarding students' beliefs and self-confidence in constructing mathematical proofs. The findings were obtained through analysis of students' written responses and semi-structured interview data. As an initial overview of students' ability to construct mathematical proofs, Table 1 presents a description of the test results based on students' engagement in deductive processes, abductive processes, and key processes.

Table 1. Description of students' mathematical proof construction test results

No	Three Proof Construction Processes (Deductive, Abductive, and Key Process) (%)	Two Proof Construction Processes (Deductive and Abductive) (%)	One Proof Construction Process (Deductive or Abductive) (%)
1	8 (19.0%)	7 (16.7%)	27 (64.3%)
	1 (2,4%)	1 (2.4%)	40 (95.2%)
	5 (11.9%)	5 (11.9%)	32 (76.2%)
2	3 (7.12%)	6 (14.3%)	33 (78.6%)
3	5 (11.9%)	1 (2.4%)	36 (85.71%

The results in the table indicate differences in students' abilities to construct mathematical proofs. In general, students were more dominant in performing deductive processes, while their ability to carry out key processes and abductive processes was relatively lower. This finding suggests that although most students were able to derive arguments deductively, only a small number were able to coherently bridge the results of deductive and abductive processes.

Based on this general overview of the test results, three students were then selected for more in-depth analysis of their beliefs and self-confidence. The students were purposively selected to represent variations in proof construction ability: a student who was able to perform all three proof construction processes (deductive, abductive, and key process), a student who was able to perform two processes (deductive and abductive), and a student who was able to perform only one proof construction process. These three students were coded as M1, M2, and M3. The subsequent analysis presents and discusses the findings in an integrated manner to provide a comprehensive description of students' affective aspects during the process of constructing mathematical proofs.

Students' Beliefs in Constructing Mathematical Proofs

The analysis of students' written responses and interview data shows that there are variations in students' levels of belief in the process of constructing mathematical proofs. These beliefs are reflected in students' ability to develop arguments, provide justification for each proof step, and connect known information to the intended conclusion through deductive, abductive, and key processes.

Student M1 demonstrated strong belief in constructing mathematical proofs. This is evident in M1's solution to Problem 1 using a contrapositive proof strategy. M1 was able to state the contrapositive correctly from the given statement to be proven. M1 also applied properties of numbers logically. Starting from the assumption that 2n + 8 is not divisible by 4, M1 manipulated the algebraic expression into the form 2(n + 4) and used the concept of multiples to conclude that n is an odd number. This step indicates that M1 did not merely rely on procedures but understood the mathematical reasoning behind each step of the proof. Furthermore, M1 connected the fact that n is an odd integer to the expression 3n + 1. The explanation that the product of two odd numbers is odd, and that adding 1 to an odd number produces an even number, shows a firm belief in the validity of the argument used.

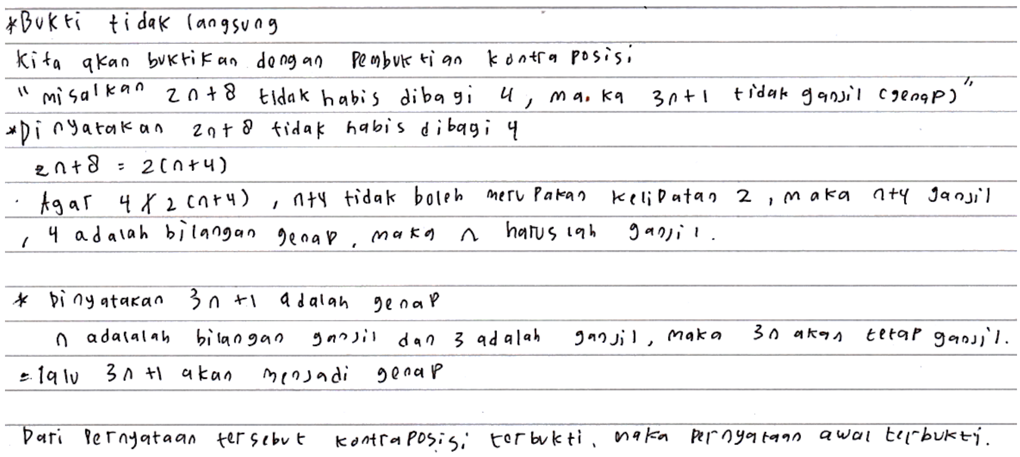
*Bukti tidak langsung
Kita akan buktikan dengan pembuktian kontra posisi
" misalkan 2n+8 tidak habis dibagi 4, maka 3n+1 tidak ganjil (genap)"
*Dinyatakan 2n+8 tidak habis dibagi 4
2n+8 = 2(n+4)
· Agar $4 \nmid 2(n+4)$, n+4 tidak boleh merupakan kelipatan 2, maka n+4 ganjil
, 4 adalah bilangan genap, maka n haruslah ganjil.
* Dinyatakan 3n+1 adalah genap
n adalah bilangan ganjil dan 3 adalah ganjil, maka 3n akan tetap ganjil.
= lalu 3n+1 akan menjadi genap
Dari pernyataan tersebut kontraposisi terbukti, maka pernyataan awal terbukti.

Figure 1. M1's answer to Problem 1

https://doi.org/10.30606/absis.v9i1.4230

Translation

Indirect Proof

We will prove it using proof by contraposition:

"Suppose $2n + 8$ *is not divisible by 4, then* $3n + 1$ *is not odd, or in other words, it is even."*

First, state that $2n + 8$ *is not divisible by 4.*

$2n + 8 = 2(n + 4)$

For 4 *not to divide* $2(n + 4)$, $n + 4$ *must not be a multiple of* 2. *Therefore,* $n + 4$ *is odd.*

Since 4 is an even number, n *must be odd.*

Next, state that $3n + 1$ *is even.*

Since n *is odd and 3 is also odd,* $3n$ *remains odd.*

Thus,

$3n + 1$

is even.

From this statement, the contrapositive has been proven. Therefore, the original statement is proven.

M1 also showed a high level of belief in solving Problem 3 (Figure 2). M1's belief in the deductive process appeared from the beginning when he deliberately chose a proof by contradiction strategy and stated the initial assumption that $x = \sqrt{5}$ is rational in lowest terms, namely $x = \frac{p}{q}$ with *gcd(p,q)* = 1. This decision indicates that M1 did not simply follow a procedure but understood the conceptual reason why the fraction must be in simplest form.

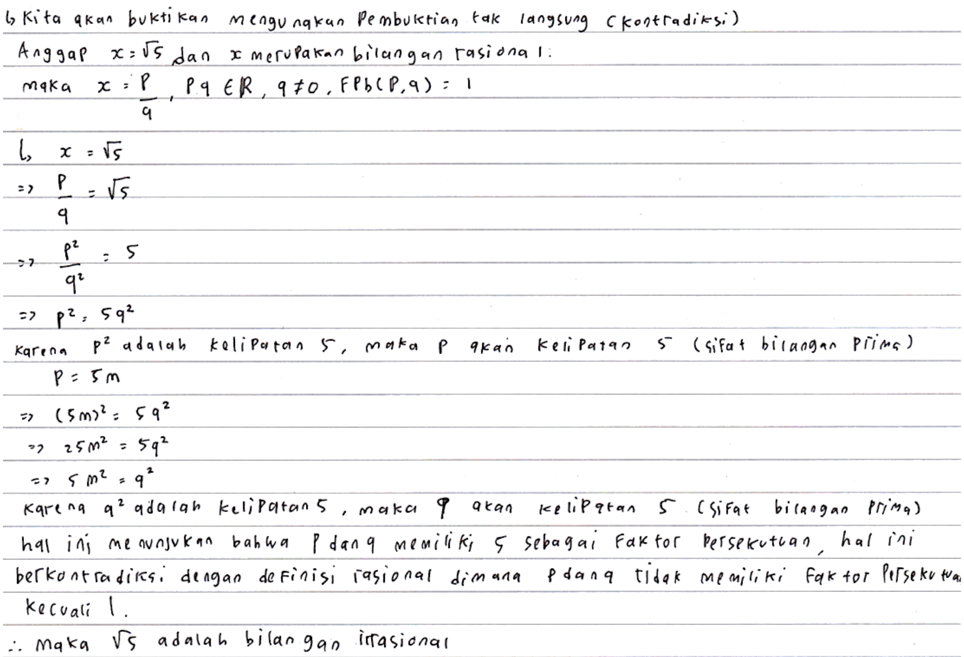
↳ Kita akan buktikan menggunakan pembuktian tak langsung (kontradiksi)
Anggap $x = \sqrt{5}$ dan x merupakan bilangan rasional:
maka $x = \frac{p}{q}$, $p, q \in \mathbb{R}$, $q \neq 0$, FPB$(p,q) = 1$
↳ $x = \sqrt{5}$
$\Rightarrow \frac{p}{q} = \sqrt{5}$
$\Rightarrow \frac{p^2}{q^2} = 5$
$\Rightarrow p^2 = 5q^2$
Karena p^2 adalah kelipatan 5, maka p akan kelipatan 5 (sifat bilangan prima)
$p = 5m$
$\Rightarrow (5m)^2 = 5q^2$
$\Rightarrow 25m^2 = 5q^2$
$\Rightarrow 5m^2 = q^2$
Karena q^2 adalah kelipatan 5, maka q akan kelipatan 5 (sifat bilangan prima)
hal ini menunjukkan bahwa p dan q memiliki 5 sebagai faktor persekutuan, hal ini berkontradiksi dengan definisi rasional dimana p dan q tidak memiliki faktor persekutuan kecuali 1.
∴ maka $\sqrt{5}$ adalah bilangan irasional

Figure 2. M1's answer to Problem 3

https://doi.org/10.30606/absis.v9i1.4230

Translation

We will prove this using an indirect proof, namely proof by contradiction.

Assume that $x = \sqrt{5}$ *and that* x *is a rational number.*

Then,

$x = \frac{p}{q}, p, q \in \mathbb{Z}, q \neq 0, gcd\ (p, q) = 1$

$x = \sqrt{5},$

$\frac{p}{q} = \sqrt{5}.$

$\frac{p^2}{q^2} = 5.$

$p^2 = 5q^2.$

Since p^2 *is a multiple of 5,* p *must also be a multiple of 5, based on the property of prime numbers.*

Let

$p = 5m.$

$(5m)^2 = 5q^2,$

$25m^2 = 5q^2.$

$5m^2 = q^2.$

Since q^2 *is a multiple of 5,* q *must also be a multiple of 5, based on the property of prime numbers.*

This shows that both p *and* q *have 5 as a common factor. This contradicts the definition of a rational number in which* p *and* q *have no common factor except 1.*

Therefore,

$\sqrt{5}$

is an irrational number.

M1 then carried out appropriate deductive steps by squaring both sides and obtaining p2 = 5q2, and used the property of prime numbers to conclude that if 5 divides p2, then 5 divides p. M1's belief in the abductive process is evident when he recognized an impossible condition—namely, the emergence of a common factor 5 in both p and q, which contradicts the initial assumption that gcd(p,q) = 1. M1 consciously directed the proof toward reaching this contradiction as an intermediate target, indicating a clear belief that a proof by contradiction must lead to a logical inconsistency. In addition, M1 demonstrated belief in the key process that bridges the deductive and abductive targets. This key process appears in the substitution step p = 5m into the equation p2 = 5q2, which yields 5m2 = q2. From this result, M1 concluded that 5 divides q. This substitution reflects M1's belief that the step is valid and relevant for reaching the intended contradiction.

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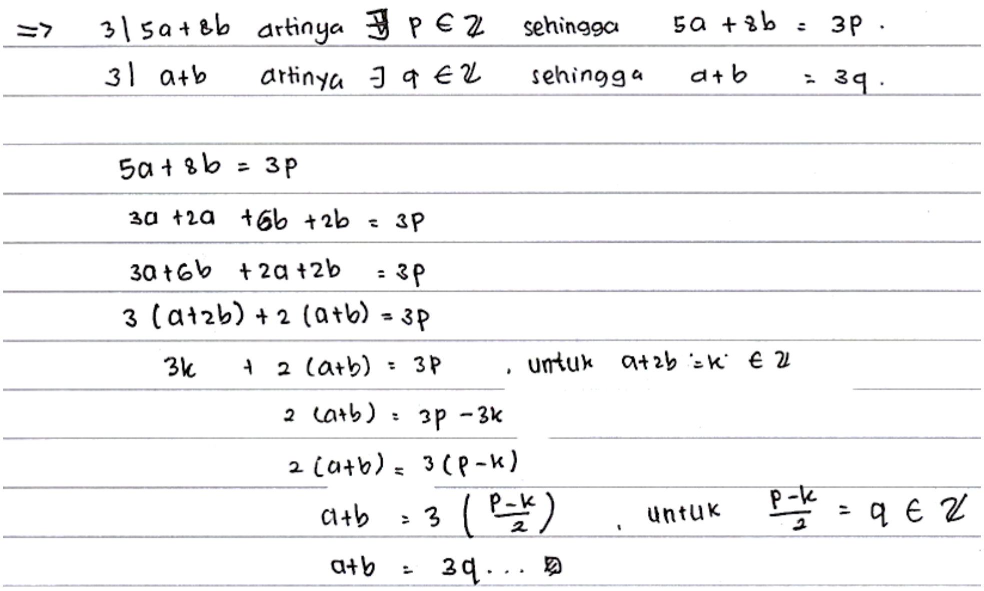
In contrast to M1, student M2 demonstrated a fairly good level of belief, but it was limited to two proof construction processes, namely the deductive and abductive processes. This can be seen in M2's work on Problem 2 (Figure 3). M2 showed reasonably strong belief in constructing a mathematical proof, particularly at the initial stage. This belief appears in the deductive process when M2 correctly began the proof using the definition of divisibility by assuming that 5a + 8b = 3p for some p ∈ Z. M2 then deductively decomposed and regrouped the algebraic terms to relate the expression 5a + 8b to the form a + b. This step indicates that M2 had confidence in the algebraic procedures used and understood the intended direction of the proof.

In addition, M2 also demonstrated belief in the abductive process by formulating an intermediate target considered most likely to lead to the final conclusion, namely by stating a + b = 3q. This intermediate target shows that M2 recognized the expected form of the conclusion and attempted to align the previous steps toward that result. However, in the key process that functions to bridge the deductive and abductive results, M2's belief was not accompanied by sufficient carefulness in the applied steps. This is evident when M2 directly divided the equation 2(a + b) = 3(p − k) by 2 and concluded that a + b = 3((p − k)/2) without first ensuring that p − k is an even integer. This inaccuracy was also revealed in the interview, when M2 was unable to explain whether (p − k)/2 must necessarily be an integer.

This inaccuracy indicates that although M2 had belief in the initial proof steps, that belief was not supported by sufficient rigor in verifying the validity of the mathematical argument. Thus, M2 demonstrated fairly good belief in the deductive and abductive processes but was not yet able to sustain that belief in the key process, resulting in a proof construction that did not form a mathematically complete proof structure.


$\Rightarrow$ $3 \mid 5a+8b$ artinya $\exists\, p \in \mathbb{Z}$ sehingga $5a+8b = 3p$.
$3 \mid a+b$ artinya $\exists\, q \in \mathbb{Z}$ sehingga $a+b = 3q$.

$5a+8b = 3p$
$3a+2a+6b+2b = 3p$
$3a+6b+2a+2b = 3p$
$3(a+2b)+2(a+b) = 3p$
$3k + 2(a+b) = 3p$, untuk $a+2b = k \in \mathbb{Z}$
$2(a+b) = 3p-3k$
$2(a+b) = 3(p-k)$
$a+b = 3\left(\frac{p-k}{2}\right)$, untuk $\frac{p-k}{2} = q \in \mathbb{Z}$
$a+b = 3q \ldots \blacksquare$

Figure 3. M2's answer to Problem 2

Translation

3|5a+8b *means that there exists* $p \in \mathbb{Z}$ *such that* $5a + 8b = 3p$.
3 ∣ $a + b$ *means that there exists* $q \in \mathbb{Z}$ *such that* $a + b = 3q$.
Starting from
$5a + 8b = 3p,$
$3a + 2a + 6b + 2b = 3p.$
$3a + 6b + 2a + 2b = 3p.$
$3(a + 2b) + 2(a + b) = 3p.$
$a + 2b = k, k \in \mathbb{Z}.$
$3k + 2(a + b) = 3p.$
$2(a + b) = 3p - 3k.$
$2(a + b) = 3(p - k).$
$a + b = 3\left(\frac{p-k}{2}\right).$
$\frac{p-k}{2} = q, q \in \mathbb{Z}.$
$a + b = 3q.$
3 ∣ $a + b$.

Meanwhile, student M3 demonstrated the most limited level of belief in constructing mathematical proofs. M3 showed belief in only one proof construction process, namely the abductive process. M3 was able to state an intermediate target derived from the final goal to be proven, but was not able to deduce it logically from the given initial conditions. M3's lack of belief in the deductive process was evident from the beginning, when M3 stated that he did not clearly understand what needed to be proven in Problem 4 (Figure 3). Although M3 was able to write the form n = 2k + 1 based on the information that n is an odd integer, he did not demonstrate belief in using the other given information, namely 3 ∤ n. This is reflected in M3's statement that he did not know how the condition 3 ∤ n was related to the final target of the proof. In addition, M3 was unable to express the general forms n = 3p + 1 or n = 3p + 2 from the information that nnn is not divisible by 3.

On the other hand, M3 showed a reasonably good level of belief in the abductive process, as he was able to state an intermediate target corresponding to the final goal, namely by writing n2 − 1 = 24m. However, the steps proposed to reach this target were expressed with hesitation, as revealed in the interview when M3 stated that he was unsure about the steps he had taken. This indicates that the intermediate target produced through the abductive process was not fully supported by a conceptual understanding of the available information. Furthermore, M3's weak belief in the deductive process had a direct impact on his inability to carry out the key process. The absence of an intermediate target derived deductively from the condition 3 ∤ n prevented M3 from bridging the abductively obtained target with the initial conditions of the problem. This situation is evident from M3's inability to respond when asked about the general form of integers that are not divisible by 3. Thus, M3's belief in the proof construction process remained limited, and the resulting proof did not form a sound logical structure. These findings indicate that limited student belief is closely related to the inability to integrate the proof construction processes as a whole.

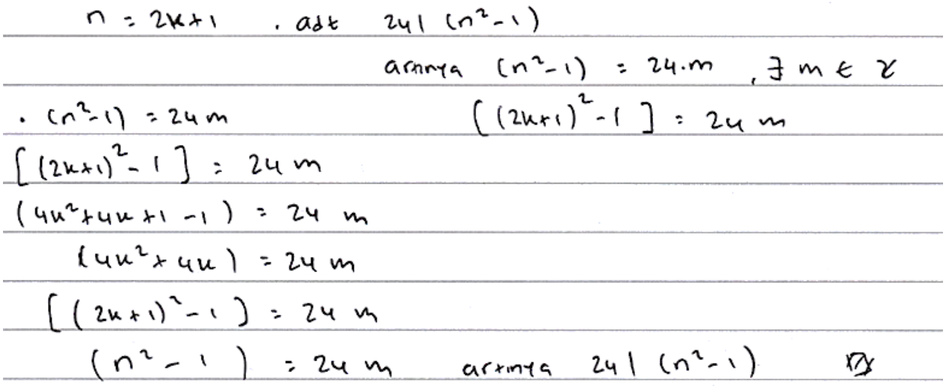

$n = 2k+1$, adt $24 \mid (n^2-1)$
artinya $(n^2-1) = 24 \cdot m$, $\exists m \in \mathbb{Z}$
$\cdot\ (n^2-1) = 24m$ $\quad [(2k+1)^2-1] = 24m$
$[(2k+1)^2-1] = 24m$
$(4k^2+4k+1-1) = 24m$
$(4k^2+4k) = 24m$
$[(2k+1)^2-1] = 24m$
$(n^2-1) = 24m$ artinya $24 \mid (n^2-1)$ ☒

Figure 4. M3's answer to Problem 4

Translation
n = 2k+1, *It will be proved that* 24 | (n2–1)
means
$(n^2 - 1) = 24m, \exists m \in \mathbb{Z}$
$(n^2 - 1) = 24m$
$[(2k + 1)^2 - 1] = 24m$
$[(2k + 1)^2 - 1] = 24m$
$(4k^2 + 4k + 1 - 1) = 24m$
$(4k^2 + 4k) = 24m$
$[(2k + 1)^2 - 1] = 24m$
$(n^2 - 1) = 24m$
meaning
$24 \mid (n^2 - 1)$.

Overall, the findings related to the belief aspect indicate that the more students are able to engage in deductive and abductive processes and carry out the key process that bridges the two, the stronger their belief in constructing mathematical proofs becomes. This condition suggests that students' belief does not arise instantly, but rather develops along with their ability to understand the logical relationships among proof steps. Students who are able to integrate these three processes tend to demonstrate more stable belief because each step of the proof is supported by conceptually understood justification. This finding is consistent with the view of Viteri & DeDeo (2022), who state that belief in a mathematical proof is not built solely through deductive reasoning, but through the interaction between deductive and abductive reasoning in the process of proof acceptance. Conversely, students' limitations in one or more proof construction processes lead to weaker belief in providing mathematical justification. In addition, the results of this study indicate that students' subjectively perceived belief is not always aligned with the quality of the proof produced, so students may feel confident about their proof process even when the arguments constructed are not yet mathematically valid (Shongwe, 2025).

Students' Self-Confidence in the Proof Construction Process

Students' self-confidence in constructing mathematical proofs was identified primarily through interviews, in which students were asked to explain and defend the proof construction processes they had developed. The interview results show that students' self-confidence does not always align with the quality of the written proofs they produced.

Student M1 demonstrated high self-confidence during the interview when explaining the proof construction process. M1 was able to describe each step of the proof in a coherent, firm, and doubt-free manner, from the initial assumption to the final conclusion. In responding to the researcher's questions, M1 not only described the proof procedures but also provided justification for each step taken, such as the reason for using proof by contradiction and the choice of representing a rational number in lowest terms with gcd(p,q) = 1.

M1's self-confidence became even more evident when responding to clarification questions from the researcher. M1 answered reflectively and argumentatively, for example by explaining that the condition gcd(p,q) = 1 is not an absolute requirement but is chosen to simplify the proof process. This indicates that M1 did not merely memorize proof steps but understood the process conceptually. Moreover, M1 maintained consistent self-confidence through the final stage of the proof. M1 confidently concluded that a contradiction arises when 5 divides both p and q, and firmly stated that the initial assumption is false and therefore $\sqrt{5}$ is irrational. Based on the interview results, M1's self-confidence is reflected not only in the willingness to present answers but also in the ability to defend arguments grounded in the constructed proof process.

Student M2 frequently expressed a lack of self-confidence in constructing mathematical proofs. At the initial stage of the proof, M2 appeared fairly confident when explaining the early steps, namely by stating 3n + 1 = 2k + 1 and explaining the goal of determining the value of nnn. This indicates that M2 had initial confidence in the chosen step and was willing to express his reasoning. However, this confidence was not consistent throughout the proving process. When the researcher questioned whether the form n = $\frac{2k}{3}$ satisfied the requirement that n be an integer, M2 admitted that he had not paid close attention to that condition. This response reveals doubt and a lack of confidence in re-evaluating previously taken steps. When further asked whether this form was problematic, M2 did not provide additional explanation, indicating a decline in confidence in defending his argument. M2's self-confidence appeared increasingly limited when the researcher criticized the step of substituting the value of n into the very statement being proven. M2 responded

 https://doi.org/10.30606/absis.v9i1.4230

briefly and accepted the correction without attempting to defend or independently revise the argument. This suggests that M2 tended to rely on the researcher's guidance and was not yet fully confident in the proof construction process he carried out.

In contrast to M1 and M2, student M3 demonstrated self-confidence that was aligned with reaching an answer quickly—although incorrect—in the process of constructing a mathematical proof. This is evident when M3 immediately modeled the statement "3n + 1 is odd" into the form 3n + 1 = 2k + 1 and then derived $n = \left(\frac{2}{3}\right)k$ without showing any doubt about the validity of this step. This confidence appeared even though the resulting form does not satisfy the requirement that nnn be an integer. M3's self-confidence is also reflected in the tendency to continue the proof process toward the intended target, namely showing that 2n + 8 is divisible by 4. M3 stated firmly that expressing 2n + 8 as "4 times something" was the goal of the proof and agreed when the researcher reaffirmed this target. However, this confidence was not accompanied by a critical evaluation of whether the steps taken were consistent with the initial assumptions and the relevant number properties.

Moreover, when the researcher pointed out that the form $n = \left(\frac{2}{3}\right)k$ is inappropriate because nnn must be an integer, M3 did not show reflection or attempt to revise the argument independently. This indicates that M3's self-confidence was based more on the belief that the steps taken were leading to an answer rather than on an understanding of a valid proof process. Thus, M3's self-confidence is characterized by a strong sense of certainty from the beginning through the end of the process, even though the proof procedure used was not mathematically valid.

Overall, the findings indicate that students' self-confidence in constructing mathematical proofs is not always directly proportional to the accuracy and completeness of the proof structure they produce. Some students are able to explain proof steps coherently and defend their arguments conceptually, while others display a high level of confidence even when the proof procedures they use are incorrect. On the other hand, there are also students who understand part of the proof steps but frequently express doubt and lack of confidence when asked to evaluate or defend their arguments. This suggests that self-confidence in proving should not be understood merely as fluency in explaining an answer, but also as the ability to reflect on and validate proof steps.

In the literature, self-confidence in mathematical tasks is often discussed through the concept of self-efficacy, defined as an individual's belief in their ability to understand and complete specific tasks. In the context of proof, self-efficacy includes belief in one's ability to understand, construct, and present proofs, as well as belief in the correctness of one's own constructed proofs (Viholainen et al., 2019). Research by Viholainen et al. (2019) shows that students may have high motivation toward proof but still demonstrate lower levels of self-belief in independently constructing and evaluating proofs, especially when their conceptual understanding and proof experience are still limited. This pattern is consistent with the findings of the present study, in which some students showed hesitation when their proof steps were criticized and tended to rely on external guidance.

These findings also indicate that feeling certain about an argument is not always identical to its formal validity. Studies on students' perceptions of proof show that confidence in an argument and judgments of formal validity may develop through different mechanisms, so a person may feel convinced by an argument that is not yet mathematically valid (Weber, 2010). This condition appears in students who quickly express confidence in their chosen steps and believe they have reached the proof goal, even though those steps do not satisfy logical validity requirements.

Other studies also report that self-efficacy plays a positive role in successful proof task completion and in the reasoning strategies used (Shimizu, 2022). Confidence in mathematical ability does correlate with performance, but it still requires conceptual understanding so that it does not develop into misplaced confidence (Parsons et al., 2009). In the context of this study, more stable self-confidence appears in students who are able to coherently integrate deductive, abductive, and key processes, so that their sense of certainty is supported by a well-understood structure of justification.

Thus, self-confidence in constructing mathematical proofs should be understood as confidence grounded in conceptual understanding, the ability to evaluate steps critically, and awareness of argument validity. Instruction in mathematical proof should therefore be designed not only to improve procedural accuracy but also to foster reflective self-confidence through activities involving justification, proof evaluation, and independent argument revision.

CONCLUSIONS AND RECOMMENDATIONS

This study aimed to examine students' beliefs and self-confidence in constructing mathematical proofs through the analysis of written responses and semi-structured interviews. The findings show that students' beliefs and self-confidence in constructing mathematical proofs vary and are closely related to the completeness of the proof construction processes they perform, particularly their engagement in deductive, abductive, and key processes.

Regarding the belief aspect, students demonstrated three categories of behavior: belief reflected in performing one, two, or all three proof construction processes. Students who were able to integrate all three processes tended to show stronger belief in providing justification and in linking proof steps logically. Conversely, limitations in one or more proof construction processes led to weaker belief in developing coherent proof arguments.

In terms of self-confidence, students also demonstrated three behavioral categories: self-confidence associated with reaching answers quickly despite being incorrect;

 https://doi.org/10.30606/absis.v9i1.4230

self-confidence characterized by frequent expressions of doubt in constructing proofs; and self-confidence reflected in the ability to explain and defend the proof construction process coherently and convincingly. These findings confirm that healthy self-confidence in mathematical proof is not determined merely by boldness or speed in reaching an answer, but by conceptual understanding and the correctness of the mathematical reasoning underlying the proof process.

Overall, this study shows that affective aspects—namely belief and self-confidence—are inseparable from cognitive processes in mathematical proof. The development of students' proof construction ability needs to address affective aspects in addition to cognitive ones. Based on these findings, future research is recommended to further investigate the relationship between affective aspects and mathematical proof construction processes by involving more diverse participants or different mathematical content areas. Further studies may also explore how specific instructional interventions can facilitate the sustained development of students' beliefs and self-confidence in learning mathematical proof.

ACKNOWLEDGMENT

The authors would like to express their gratitude to the Mathematics Education Study Program at Universitas Pendidikan Indonesia for providing facility support during the implementation of this research. The authors also thank the mathematics education experts who contributed to the validation of the research instruments, as well as the students who participated in this study.

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