

## AN EXPLORATION OF STUDENTS' BELIEFS AND SELF-CONFIDENCE IN CONSTRUCTING MATHEMATICAL PROOFS

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**ABSTRACT** The ability to construct mathematical proofs is one of the essential competencies in mathematics learning at the university level. Understanding the process of constructing mathematical proofs cannot be achieved solely through cognitive aspects, but also requires attention to students' affective aspects. This study aims to examine affective aspects in the form of students' beliefs and self-confidence in constructing mathematical proofs. The study employed a qualitative approach with a case study design. The research subjects were 42 second-semester students at a public university in Indonesia who had completed a Number Theory course. Data were collected through a written test consisting of mathematical proof problems and semi-structured interviews. For in-depth analysis, three students were purposively selected to represent variations in the mathematical proof construction process. Data analysis was conducted using a descriptive qualitative method by examining students' beliefs and self-confidence during the process of constructing mathematical proofs. The results show that students' beliefs can be classified into three categories based on each proof construction process they performed. In addition, students' self-confidence is also grouped into three categories, ranging from confidence associated with reaching answers quickly despite being incorrect, to confidence demonstrated through the ability to explain and defend the proof construction process in a coherent and convincing manner. These findings emphasize the important role of affective aspects, in addition to cognitive aspects, in understanding students' behavior in constructing mathematical proofs.

**Keywords:** mathematical proof, belief, self-confidence, deductive process, abductive process

**ABSTRAK** Kemampuan mengonstruksi bukti matematis merupakan salah satu kompetensi penting dalam pembelajaran matematika di perguruan tinggi. Pemahaman terhadap proses konstruksi bukti matematis tidak dapat dicapai hanya melalui kajian aspek kognitif, tetapi juga memerlukan perhatian terhadap aspek afektif mahasiswa. Penelitian ini bertujuan untuk mengkaji aspek afektif berupa keyakinan dan kepercayaan diri mahasiswa dalam mengonstruksi bukti matematis. Penelitian menggunakan pendekatan kualitatif dengan desain studi kasus. Subjek penelitian adalah 42 mahasiswa semester II pada sebuah universitas negeri di Indonesia yang telah menempuh mata kuliah Teori Bilangan. Data dikumpulkan melalui tes tertulis berupa soal pembuktian matematis dan wawancara semi-

terstruktur. Untuk pendalaman analisis, dipilih tiga mahasiswa secara purposif yang mewakili variasi proses konstruksi bukti matematis. Analisis data dilakukan secara deskriptif-kualitatif dengan menelaah keyakinan dan kepercayaan diri mahasiswa selama proses mengonstruksi bukti matematis. Hasil penelitian menunjukkan bahwa keyakinan mahasiswa terbagi ke dalam tiga kategori berdasarkan setiap proses konstruksi bukti yang dilakukan. Selain itu, kepercayaan diri mahasiswa juga terbagi ke dalam tiga kategori, mulai dari kepercayaan diri yang sejalan dengan pencapaian jawaban secara cepat meskipun tidak tepat hingga kepercayaan diri yang ditunjukkan melalui kemampuan menjelaskan dan mempertahankan proses konstruksi bukti secara runtut dan meyakinkan. Temuan ini menegaskan pentingnya peran aspek afektif, selain aspek kognitif, dalam memahami perilaku mahasiswa dalam mengonstruksi bukti matematis.

**Kata-kata kunci:** pembuktian matematis, keyakinan, kepercayaan diri, proses deduktif, proses abduktif

## INTRODUCTION

Mathematical proof is one of the fundamental activities in mathematics (CadwalladerOlsker, 2011; Hanna, 2000) and plays an important role in the development of students' reasoning, conceptual understanding, and mathematical ways of thinking. Through proof, students not only verify the truth of a mathematical statement (CadwalladerOlsker, 2011; Hersh, 1993; Tall, 1998), but also learn to connect definitions, properties, and theorems in a logical and systematic manner (Selden & Selden, 2008). Therefore, the ability to construct mathematical proofs is an essential competence that must be mastered by students, particularly at the university level (Weber, 2001).

Various studies show that students frequently experience difficulties in constructing mathematical proofs. These difficulties include an inability to understand the statement to be proven (Selden & Selden, 2008), errors in using definitions and theorems (Azrou, 2013; Doruk & Kaplan, 2015; Noto et al., 2019), and failure to organize logical arguments (Selden et al., 2014). Most research on mathematical proof tends to emphasize cognitive aspects, such as reasoning errors, proof strategies, and the structure of arguments developed by students. This approach has made important contributions to understanding how students think when developing mathematical proofs.

However, students' understanding of the process of constructing mathematical proofs cannot be achieved by focusing solely on cognitive aspects. The proving process also involves affective aspects (Furinghetti & Morselli, 2009), such as belief in the steps taken and self-confidence in explaining and defending mathematical arguments. These affective aspects influence how students begin a proof, make decisions in selecting strategies, and persist when facing doubt or difficulty during the proving process. Students' beliefs about proof and about themselves as learners of proof are also known to play a role in shaping their mathematical behavior when constructing and evaluating proofs (Stylianou et al., 2015). Students with low belief and self-confidence tend to doubt the steps they take or rely on procedures without

understanding the underlying mathematical reasons, whereas students with excessive self-confidence may reach conclusions quickly even when their arguments are not valid.

Several studies have examined affective aspects in mathematical proof (Furinghetti & Morselli, 2009; Selden & Selden, 2013; Viholainen et al., 2019). Nevertheless, studies that specifically investigate students' beliefs and self-confidence in relation to the process of constructing mathematical proofs are still relatively limited. In addition, few studies link these affective aspects to students' engagement in proof construction processes, such as deductive processes, abductive processes, and key processes that bridge both in building a complete proof argument.

In this study, the process of constructing mathematical proofs is viewed as involving three main components: the deductive process, the abductive process, and the key process (Kusnandi, 2008). The deductive process relates to deriving arguments logically from known information, the abductive process relates to formulating conjectures or intermediate targets that enable the conclusion to be reached, while the key process functions to bridge the results of deductive and abductive processes so that a coherent proof structure is formed. Students' success in integrating these three processes reflects not only cognitive ability but is also related to belief and self-confidence, which influence how students behave throughout the proving process (Shongwe, 2025).

Based on this background, this study aims to examine students' beliefs and self-confidence in constructing mathematical proofs by analyzing their engagement in deductive, abductive, and key processes. This study is expected to contribute to a more comprehensive understanding of the role of affective aspects in mathematical proof and to serve as a basis for developing proof-oriented instruction that emphasizes not only cognitive aspects but also the development of students' affective aspects.

## **METHODS**

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This study employed a qualitative approach with a case study design to obtain an in-depth understanding of students' beliefs and self-confidence in constructing mathematical proofs (Crowe et al., 2011). This approach was selected because it enables researchers to examine comprehensively the process of mathematical proof construction along with the accompanying affective aspects. The study consisted of three stages: the preparation stage, the data collection stage, and the data analysis stage.

In the first stage, the researchers conducted a literature review related to mathematical proof and affective aspects, particularly students' beliefs and self-confidence in the proof construction process. This review served as a conceptual foundation for formulating analytical indicators and developing the research instruments. The main instrument consisted of four mathematical proof problems in

basic number theory that required students to independently construct formal proofs. These problems were validated by two mathematics education experts to ensure content relevance, appropriate difficulty level, and their potential to elicit variations in proof construction processes and students' affective responses, as presented in Table 1.

**Table 1.** Problem statement

Prove the following statement using two proof strategies (direct and indirect proof): "Let $n \in \mathbb{Z}$ . If $3n + 1$ is odd, then $2n + 8$ is divisible by 4."
Let $a, b \in \mathbb{Z}$ . Prove that $3 \mid (5a + 8b)$ if and only if $3 \mid (a + b)$ .
Prove that the real number $\sqrt{5}$ is irrational.
Let $n$ be an odd integer and $3 \nmid n$ . Show that $24 \mid (n^2 - 1)$ .

The second stage was the data collection stage. Data were collected through an individual written test administered for 60 minutes and completed by 42 second-semester students from one class at a public university in Bandung, Indonesia. All students had completed a Number Theory course and were therefore assumed to possess sufficient prior mathematical knowledge to construct mathematical proofs. In this test, students were asked to write a complete proof construction process for each given mathematical statement on the provided answer sheets. During the test, students were not allowed to use smartphones, calculators, or any other electronic devices, so that the proof construction process reflected each student's individual ability.

To obtain a deeper understanding of students' affective aspects, semi-structured interviews were also conducted at this stage. Based on the results of the written test, three students were purposively selected for in-depth analysis and interviews, representing variations in their proof construction processes: a student who was able to perform all three proof construction processes (deductive, abductive, and key process), a student who was able to perform two processes (deductive and abductive), and a student who was able to perform only one process (either deductive or abductive). The interviews were conducted after the written test and focused on exploring students' beliefs regarding the proof steps they used, as well as their self-confidence when explaining and defending their proof construction process.

In this study, students' beliefs in constructing mathematical proofs were identified through their ability to provide arguments and justifications in each stage of the construction process. Meanwhile, students' self-confidence was identified through their verbal explanations during the interviews, as reflected in the fluency of their explanations, their willingness to defend their arguments, and their responses to questions or doubts raised about the proofs they proposed.

The third stage was data analysis. The analyzed data consisted of students' written responses and interview transcripts. The analysis was conducted using a descriptive qualitative approach by examining in depth the characteristics of students' beliefs and self-confidence during the process of constructing mathematical proofs. To enhance the validity of the findings, source and method triangulation were applied by comparing written data and interview data. The results of the analysis were then synthesized to provide a comprehensive description of variations in students' beliefs and self-confidence in constructing mathematical proofs.

## FINDING AND DISCUSSION

This section presents and discusses the research findings regarding students' beliefs and self-confidence in constructing mathematical proofs. The findings were obtained through analysis of students' written responses and semi-structured interview data. As an initial overview of students' ability to construct mathematical proofs, Table 1 presents a description of the test results based on students' engagement in deductive processes, abductive processes, and key processes.

**Table 1.** Description of students' mathematical proof construction test results

Problem No.	Mathematical Proof Construction Process		
	Deductive Process	Key Process	Abductive Process
1	11 (26.2%)	23 (54.8%)	8 (19%)
2	41 (97.6%)	0 (0%)	1 (2,4%)
3	37 (88%)	0 (0%)	5 (12%)
4	31 (73.8%)	8 (19%)	3 (7.2%)
Percentage	71.43%	18.45%	10.12%

The results in the table indicate differences in students' abilities to construct mathematical proofs. In general, students were more dominant in performing deductive processes, while their ability to carry out key processes and abductive processes was relatively lower. This finding suggests that although most students were able to derive arguments deductively, only a small number were able to coherently bridge the results of deductive and abductive processes.

Based on this general overview of the test results, three students were then selected for more in-depth analysis of their beliefs and self-confidence. The students were purposively selected to represent variations in proof construction ability: a student who was able to perform all three proof construction processes (deductive, abductive, and key process), a student who was able to perform two processes (deductive and abductive), and a student who was able to perform only one proof construction process. These three students were coded as M1, M2, and M3. The subsequent analysis presents and discusses the findings in an integrated manner to

provide a comprehensive description of students' affective aspects during the process of constructing mathematical proofs.

## Students' Beliefs in Constructing Mathematical Proofs

The analysis of students' written responses and interview data shows that there are variations in students' levels of belief in the process of constructing mathematical proofs. These beliefs are reflected in students' ability to develop arguments, provide justification for each proof step, and connect known information to the intended conclusion through deductive, abductive, and key processes.

Student M1 demonstrated strong belief in constructing mathematical proofs. This is evident in M1's solution to Problem 1 using a contrapositive proof strategy. M1 was able to state the contrapositive correctly from the given statement to be proven. M1 also applied properties of numbers logically. Starting from the assumption that  $2n + 8$  is not divisible by 4, M1 manipulated the algebraic expression into the form  $2(n + 4)$  and used the concept of multiples to conclude that  $n$  is an odd number. This step indicates that M1 did not merely rely on procedures but understood the mathematical reasoning behind each step of the proof. Furthermore, M1 connected the fact that  $n$  is an odd integer to the expression  $3n + 1$ . The explanation that the product of two odd numbers is odd, and that adding 1 to an odd number produces an even number, shows a firm belief in the validity of the argument used.

\*Bukti tidak langsung  
Kita akan buktikan dengan pembuktian kontraposisi  
"misalkan  $2n + 8$  tidak habis dibagi 4, maka  $3n + 1$  tidak ganjil (genap)"  
\*Dinyatakan  $2n + 8$  tidak habis dibagi 4  
 $2n + 8 = 2(n + 4)$   
• Agar  $4 \nmid 2(n + 4)$ ,  $n + 4$  tidak boleh merupakan kelipatan 2, maka  $n + 4$  ganjil  
, 4 adalah bilangan genap, maka  $n$  haruslah ganjil.  
\*Dinyatakan  $3n + 1$  adalah genap  
 $n$  adalah bilangan ganjil dan 3 adalah ganjil, maka  $3n$  akan tetap ganjil.  
=itu  $3n + 1$  akan menjadi genap  
Dari pernyataan tersebut kontraposisi terbukti, maka pernyataan awal terbukti.

Figure 1. M1's answer to Problem 1

### Translation

#### Indirect Proof

We will prove it using proof by contraposition:

"Suppose  $2n + 8$  is not divisible by 4, then  $3n + 1$  is not odd, or in other words, it is even."

First, state that  $2n + 8$  is not divisible by 4.

$$2n + 8 = 2(n + 4)$$

For 4 not to divide  $2(n + 4)$ ,  $n + 4$  must not be a multiple of 2. Therefore,  $n + 4$  is odd.

Since 4 is an even number,  $n$  must be odd.

Next, state that  $3n + 1$  is even.

Since  $n$  is odd and 3 is also odd,  $3n$  remains odd.

Thus,

$$3n + 1$$

is even.

From this statement, the contrapositive has been proven. Therefore, the original statement is proven.

M1 also showed a high level of belief in solving Problem 3 (Figure 2). M1's belief in the deductive process appeared from the beginning when he deliberately chose a proof by contradiction strategy and stated the initial assumption that  $x = \sqrt{5}$  is rational in lowest terms, namely  $x = \frac{p}{q}$  with  $\gcd(p, q) = 1$ . This decision indicates that M1 did not simply follow a procedure but understood the conceptual reason why the fraction must be in simplest form.

↳ Kita akan buktikan menggunakan Pembuktian tak langsung (kontradiksi)

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Anggap  $x = \sqrt{5}$  dan  $x$  merupakan bilangan rasional:

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maka  $x = \frac{p}{q}$ ,  $p, q \in \mathbb{R}$ ,  $q \neq 0$ ,  $\text{FPB}(p, q) = 1$

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↳  $x = \sqrt{5}$

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$\Rightarrow \frac{p}{q} = \sqrt{5}$

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$\Rightarrow \frac{p^2}{q^2} = 5$

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$\Rightarrow p^2 = 5q^2$

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karena  $p^2$  adalah kelipatan 5, maka  $p$  akan kelipatan 5 (sifat bilangan prima)

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$p = 5m$

---

$\Rightarrow (5m)^2 = 5q^2$

---

$\Rightarrow 25m^2 = 5q^2$

---

$\Rightarrow 5m^2 = q^2$

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karena  $q^2$  adalah kelipatan 5, maka  $q$  akan kelipatan 5 (sifat bilangan prima)

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hal ini menunjukkan bahwa  $p$  dan  $q$  memiliki 5 sebagai faktor persekutuan, hal ini berkontradiksi dengan definisi rasional dimana  $p$  dan  $q$  tidak memiliki faktor persekutuan kecuali 1.

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$\therefore$  maka  $\sqrt{5}$  adalah bilangan irasional

Figure 2. M1's answer to Problem 3

### Translation

We will prove this using an indirect proof, namely proof by contradiction.

Assume that  $x = \sqrt{5}$  and that  $x$  is a rational number.

Then,

$$x = \frac{p}{q}, p, q \in \mathbb{Z}, q \neq 0, \gcd(p, q) = 1$$

$$x = \sqrt{5},$$

$$\frac{p}{q} = \sqrt{5}.$$

$$\frac{p^2}{q^2} = 5.$$

$$p^2 = 5q^2.$$

Since  $p^2$  is a multiple of 5,  $p$  must also be a multiple of 5, based on the property of prime numbers.

Let

$$p = 5m.$$

$$(5m)^2 = 5q^2,$$

$$25m^2 = 5q^2.$$

$$5m^2 = q^2.$$

Since  $q^2$  is a multiple of 5,  $q$  must also be a multiple of 5, based on the property of prime numbers.

This shows that both  $p$  and  $q$  have 5 as a common factor. This contradicts the definition of a rational number in which  $p$  and  $q$  have no common factor except 1.

Therefore,

$$\sqrt{5}$$

is an irrational number.

M1 then carried out appropriate deductive steps by squaring both sides and obtaining  $p^2 = 5q^2$ , and used the property of prime numbers to conclude that if 5 divides  $p^2$ , then 5 divides  $p$ . M1's belief in the abductive process is evident when he recognized an impossible condition—namely, the emergence of a common factor 5 in both  $p$  and  $q$ , which contradicts the initial assumption that  $\gcd(p, q) = 1$ . M1 consciously directed the proof toward reaching this contradiction as an intermediate target, indicating a clear belief that a proof by contradiction must lead to a logical inconsistency. In addition, M1 demonstrated belief in the key process that bridges the deductive and abductive targets. This key process appears in the substitution step  $p = 5m$  into the equation  $p^2 = 5q^2$ , which yields  $5m^2 = q^2$ . From this result, M1 concluded that 5 divides  $q$ . This substitution reflects M1's belief that the step is valid and relevant for reaching the intended contradiction.

In contrast to M1, student M2 demonstrated a fairly good level of belief, but it was limited to two proof construction processes, namely the deductive and abductive processes. This can be seen in M2's work on Problem 2 (Figure 3). M2 showed reasonably strong belief in constructing a mathematical proof, particularly at the initial stage. This belief appears in the deductive process when M2 correctly began

the proof using the definition of divisibility by assuming that  $5a + 8b = 3p$  for some  $p \in \mathbb{Z}$ . M2 then deductively decomposed and regrouped the algebraic terms to relate the expression  $5a + 8b$  to the form  $a + b$ . This step indicates that M2 had confidence in the algebraic procedures used and understood the intended direction of the proof.

In addition, M2 also demonstrated belief in the abductive process by formulating an intermediate target considered most likely to lead to the final conclusion, namely by stating  $a + b = 3q$ . This intermediate target shows that M2 recognized the expected form of the conclusion and attempted to align the previous steps toward that result. However, in the key process that functions to bridge the deductive and abductive results, M2's belief was not accompanied by sufficient carefulness in the applied steps. This is evident when M2 directly divided the equation  $2(a + b) = 3(p - k)$  by 2 and concluded that  $a + b = 3((p - k)/2)$  without first ensuring that  $p - k$  is an even integer. This inaccuracy was also revealed in the interview, when M2 was unable to explain whether  $(p - k)/2$  must necessarily be an integer.

This inaccuracy indicates that although M2 had belief in the initial proof steps, that belief was not supported by sufficient rigor in verifying the validity of the mathematical argument. Thus, M2 demonstrated fairly good belief in the deductive and abductive processes but was not yet able to sustain that belief in the key process, resulting in a proof construction that did not form a mathematically complete proof structure.

$$\begin{aligned} \Rightarrow 3 \mid 5a + 8b & \text{ artinya } \exists p \in \mathbb{Z} \text{ sehingga } 5a + 8b = 3p. \\ 3 \mid a + b & \text{ artinya } \exists q \in \mathbb{Z} \text{ sehingga } a + b = 3q. \end{aligned}$$


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$$\begin{aligned} 5a + 8b &= 3p \\ 3a + 2a + 6b + 2b &= 3p \\ 3a + 6b + 2a + 2b &= 3p \\ 3(a + 2b) + 2(a + b) &= 3p \\ 3k + 2(a + b) &= 3p, \text{ untuk } a + 2b = k \in \mathbb{Z} \\ 2(a + b) &= 3p - 3k \\ 2(a + b) &= 3(p - k) \\ a + b &= 3 \left( \frac{p - k}{2} \right), \text{ untuk } \frac{p - k}{2} = q \in \mathbb{Z} \\ a + b &= 3q \dots \end{aligned}$$

Figure 3. M2's answer to Problem 2

### Translation

$3 \mid 5a + 8b$  means that there exists  $p \in \mathbb{Z}$  such that  $5a + 8b = 3p$ .

$3 \mid a + b$  means that there exists  $q \in \mathbb{Z}$  such that  $a + b = 3q$ .

Starting from

$$5a + 8b = 3p,$$

$$3a + 2a + 6b + 2b = 3p.$$

$$3a + 6b + 2a + 2b = 3p.$$

$$3(a + 2b) + 2(a + b) = 3p.$$

$$a + 2b = k, k \in \mathbb{Z}.$$

$$3k + 2(a + b) = 3p.$$

$$2(a + b) = 3p - 3k.$$

$$2(a + b) = 3(p - k).$$

$$a + b = 3\left(\frac{p - k}{2}\right).$$

$$\frac{p - k}{2} = q, q \in \mathbb{Z}.$$

$$a + b = 3q.$$

$$3 \mid a + b.$$

Meanwhile, student M3 demonstrated the most limited level of belief in constructing mathematical proofs. M3 showed belief in only one proof construction process, namely the abductive process. M3 was able to state an intermediate target derived from the final goal to be proven, but was not able to deduce it logically from the given initial conditions. M3's lack of belief in the deductive process was evident from the beginning, when M3 stated that he did not clearly understand what needed to be proven in Problem 4 (Figure 3). Although M3 was able to write the form  $n = 2k + 1$  based on the information that  $n$  is an odd integer, he did not demonstrate belief in using the other given information, namely  $3 \nmid n$ . This is reflected in M3's statement that he did not know how the condition  $3 \nmid n$  was related to the final target of the proof. In addition, M3 was unable to express the general forms  $n = 3p + 1$  or  $n = 3p + 2$  from the information that  $nnn$  is not divisible by 3.

On the other hand, M3 showed a reasonably good level of belief in the abductive process, as he was able to state an intermediate target corresponding to the final goal, namely by writing  $n^2 - 1 = 24m$ . However, the steps proposed to reach this target were expressed with hesitation, as revealed in the interview when M3 stated that he was unsure about the steps he had taken. This indicates that the intermediate target produced through the abductive process was not fully supported by a conceptual understanding of the available information. Furthermore, M3's weak belief in the deductive process had a direct impact on his inability to carry out the key process. The absence of an intermediate target derived deductively from the condition  $3 \nmid n$  prevented M3 from bridging the abductively obtained target with the initial conditions of the problem. This situation is evident from M3's inability to respond when asked about the general form of integers that are not divisible by 3. Thus, M3's belief in the proof construction process remained limited, and the resulting proof did not form a sound logical structure. These findings indicate that limited student belief is closely related to the inability to integrate the proof construction processes as a whole.

$$\begin{array}{l}
 n = 2k+1 \quad . \text{ adk} \quad 24 | (n^2-1) \\
 \text{artinya} \quad (n^2-1) = 24 \cdot m \quad , \exists m \in \mathbb{Z} \\
 \cdot (n^2-1) = 24 m \quad \quad \quad [(2k+1)^2-1] = 24 m \\
 [(2k+1)^2-1] = 24 m \\
 (4k^2+4k+1-1) = 24 m \\
 (4k^2+4k) = 24 m \\
 [(2k+1)^2-1] = 24 m \\
 (n^2-1) = 24 m \quad \text{artinya} \quad 24 | (n^2-1) \quad \square
 \end{array}$$

Figure 4. M3's answer to Problem 4

*Translation*

$n = 2k+1$ , It will be proved that  $24 | (n^2-1)$

*means*

$$(n^2 - 1) = 24m, \exists m \in \mathbb{Z}$$

$$(n^2 - 1) = 24m$$

$$[(2k + 1)^2 - 1] = 24m$$

$$[(2k + 1)^2 - 1] = 24m$$

$$(4k^2 + 4k + 1 - 1) = 24m$$

$$(4k^2 + 4k) = 24m$$

$$[(2k + 1)^2 - 1] = 24m$$

$$(n^2 - 1) = 24m$$

*meaning*

$$24 | (n^2 - 1).$$

Overall, the findings related to the belief aspect indicate that the more students are able to engage in deductive and abductive processes and carry out the key process that bridges the two, the stronger their belief in constructing mathematical proofs becomes. This condition suggests that students' belief does not arise instantly, but rather develops along with their ability to understand the logical relationships among proof steps. Students who are able to integrate these three processes tend to demonstrate more stable belief because each step of the proof is supported by conceptually understood justification. This finding is consistent with the view of Viteri & DeDeo (2022), who state that belief in a mathematical proof is not built solely through deductive reasoning, but through the interaction between deductive and abductive reasoning in the process of proof acceptance. Conversely, students' limitations in one or more proof construction processes lead to weaker belief in providing mathematical justification. In addition, the results of this study indicate that students' subjectively perceived belief is not always aligned with the quality of the proof produced, so students may feel confident about their proof process even when the arguments constructed are not yet mathematically valid (Shongwe, 2025).

### Students' Self-Confidence in the Proof Construction Process

Students' self-confidence in constructing mathematical proofs was identified primarily through interviews, in which students were asked to explain and defend the proof construction processes they had developed. The interview results show that students' self-confidence does not always align with the quality of the written proofs they produced.

Student M1 demonstrated high self-confidence during the interview when explaining the proof construction process. M1 was able to describe each step of the proof in a coherent, firm, and doubt-free manner, from the initial assumption to the final conclusion. In responding to the researcher's questions, M1 not only described the proof procedures but also provided justification for each step taken, such as the reason for using proof by contradiction and the choice of representing a rational number in lowest terms with  $\gcd(p,q) = 1$ .

M1's self-confidence became even more evident when responding to clarification questions from the researcher. M1 answered reflectively and argumentatively, for example by explaining that the condition  $\gcd(p,q) = 1$  is not an absolute requirement but is chosen to simplify the proof process. This indicates that M1 did not merely memorize proof steps but understood the process conceptually. Moreover, M1 maintained consistent self-confidence through the final stage of the proof. M1 confidently concluded that a contradiction arises when 5 divides both  $p$  and  $q$ , and firmly stated that the initial assumption is false and therefore  $\sqrt{5}$  is irrational. Based on the interview results, M1's self-confidence is reflected not only in the willingness to present answers but also in the ability to defend arguments grounded in the constructed proof process.

Student M2 frequently expressed a lack of self-confidence in constructing mathematical proofs. At the initial stage of the proof, M2 appeared fairly confident when explaining the early steps, namely by stating  $3n + 1 = 2k + 1$  and explaining the goal of determining the value of  $n$ . This indicates that M2 had initial confidence in the chosen step and was willing to express his reasoning. However, this confidence was not consistent throughout the proving process. When the researcher questioned whether the form  $n = \frac{2k}{3}$  satisfied the requirement that  $n$  be an integer, M2 admitted that he had not paid close attention to that condition. This response reveals doubt and a lack of confidence in re-evaluating previously taken steps. When further asked whether this form was problematic, M2 did not provide additional explanation, indicating a decline in confidence in defending his argument. M2's self-confidence appeared increasingly limited when the researcher criticized the step of substituting the value of  $n$  into the very statement being proven. M2 responded briefly and accepted the correction without attempting to defend or independently revise the argument. This suggests that M2 tended to rely on the researcher's guidance and was not yet fully confident in the proof construction process he carried out.

In contrast to M1 and M2, student M3 demonstrated self-confidence that was aligned with reaching an answer quickly—although incorrect—in the process of constructing a mathematical proof. This is evident when M3 immediately modeled the statement “ $3n + 1$  is odd” into the form  $3n + 1 = 2k + 1$  and then derived  $n = \left(\frac{2}{3}\right)k$  without showing any doubt about the validity of this step. This confidence appeared even though the resulting form does not satisfy the requirement that  $n$  be an integer. M3’s self-confidence is also reflected in the tendency to continue the proof process toward the intended target, namely showing that  $2n + 8$  is divisible by 4. M3 stated firmly that expressing  $2n + 8$  as “4 times something” was the goal of the proof and agreed when the researcher reaffirmed this target. However, this confidence was not accompanied by a critical evaluation of whether the steps taken were consistent with the initial assumptions and the relevant number properties.

Moreover, when the researcher pointed out that the form  $n = \left(\frac{2}{3}\right)k$  is inappropriate because  $n$  must be an integer, M3 did not show reflection or attempt to revise the argument independently. This indicates that M3’s self-confidence was based more on the belief that the steps taken were leading to an answer rather than on an understanding of a valid proof process. Thus, M3’s self-confidence is characterized by a strong sense of certainty from the beginning through the end of the process, even though the proof procedure used was not mathematically valid.

Overall, the findings indicate that students’ self-confidence in constructing mathematical proofs is not always directly proportional to the accuracy and completeness of the proof structure they produce. Some students are able to explain proof steps coherently and defend their arguments conceptually, while others display a high level of confidence even when the proof procedures they use are incorrect. On the other hand, there are also students who understand part of the proof steps but frequently express doubt and lack of confidence when asked to evaluate or defend their arguments. This suggests that self-confidence in proving should not be understood merely as fluency in explaining an answer, but also as the ability to reflect on and validate proof steps.

In the literature, self-confidence in mathematical tasks is often discussed through the concept of self-efficacy, defined as an individual’s belief in their ability to understand and complete specific tasks. In the context of proof, self-efficacy includes belief in one’s ability to understand, construct, and present proofs, as well as belief in the correctness of one’s own constructed proofs (Viholainen et al., 2019). Research by Viholainen et al. (2019) shows that students may have high motivation toward proof but still demonstrate lower levels of self-belief in independently constructing and evaluating proofs, especially when their conceptual understanding and proof experience are still limited. This pattern is consistent with the findings of the present study, in which some students showed hesitation when their proof steps were criticized and tended to rely on external guidance.

These findings also indicate that feeling certain about an argument is not always identical to its formal validity. Studies on students' perceptions of proof show that confidence in an argument and judgments of formal validity may develop through different mechanisms, so a person may feel convinced by an argument that is not yet mathematically valid (Weber, 2010). This condition appears in students who quickly express confidence in their chosen steps and believe they have reached the proof goal, even though those steps do not satisfy logical validity requirements.

Other studies also report that self-efficacy plays a positive role in successful proof task completion and in the reasoning strategies used (Shimizu, 2022). Confidence in mathematical ability does correlate with performance, but it still requires conceptual understanding so that it does not develop into misplaced confidence (Parsons et al., 2009). In the context of this study, more stable self-confidence appears in students who are able to coherently integrate deductive, abductive, and key processes, so that their sense of certainty is supported by a well-understood structure of justification.

Thus, self-confidence in constructing mathematical proofs should be understood as confidence grounded in conceptual understanding, the ability to evaluate steps critically, and awareness of argument validity. Instruction in mathematical proof should therefore be designed not only to improve procedural accuracy but also to foster reflective self-confidence through activities involving justification, proof evaluation, and independent argument revision.

## **CONCLUSIONS AND RECOMMENDATIONS**

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This study aimed to examine students' beliefs and self-confidence in constructing mathematical proofs through the analysis of written responses and semi-structured interviews. The findings show that students' beliefs and self-confidence in constructing mathematical proofs vary and are closely related to the completeness of the proof construction processes they perform, particularly their engagement in deductive, abductive, and key processes.

Regarding the belief aspect, students demonstrated three categories of behavior: belief reflected in performing one, two, or all three proof construction processes. Students who were able to integrate all three processes tended to show stronger belief in providing justification and in linking proof steps logically. Conversely, limitations in one or more proof construction processes led to weaker belief in developing coherent proof arguments.

In terms of self-confidence, students also demonstrated three behavioral categories: self-confidence associated with reaching answers quickly despite being incorrect; self-confidence characterized by frequent expressions of doubt in constructing proofs; and self-confidence reflected in the ability to explain and defend the proof construction process coherently and convincingly. These findings confirm that healthy self-confidence in mathematical proof is not determined merely by boldness

or speed in reaching an answer, but by conceptual understanding and the correctness of the mathematical reasoning underlying the proof process.

Overall, this study shows that affective aspects—namely belief and self-confidence—are inseparable from cognitive processes in mathematical proof. The development of students' proof construction ability needs to address affective aspects in addition to cognitive ones. Based on these findings, future research is recommended to further investigate the relationship between affective aspects and mathematical proof construction processes by involving more diverse participants or different mathematical content areas. Further studies may also explore how specific instructional interventions can facilitate the sustained development of students' beliefs and self-confidence in learning mathematical proof.

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### REFERENCES

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- Azrou, N. (2013). Proof in algebra at the university level: Analysis of students' difficulties. *Proceedings of CERME 8: Eighth Congress of the European Society for Research in Mathematics Education*, 76–85.  
[http://cerme8.metu.edu.tr/wgpapers/WG1/WG1\\_Azrou.pdf](http://cerme8.metu.edu.tr/wgpapers/WG1/WG1_Azrou.pdf)
- CadwalladerOlsker, T. (2011). What do we mean by mathematical proof? *Journal of Humanistic Mathematics*, 1(1), 33–60.  
<https://doi.org/10.5642/jhummath.201101.04>
- Crowe, S., Cresswell, K., Robertson, A., Huby, G., Avery, A., & Sheikh, A. (2011). The case study approach. *BMC Medical Research Methodology*, 11(1), Article 100.  
<https://doi.org/10.1186/1471-2288-11-100>
- Doruk, M., & Kaplan, A. (2015). Prospective mathematics teachers' difficulties in doing proofs and causes of their struggle with proofs. *Bayburt Eğitim Fakültesi Dergisi*, 10(2), 315–328.
- Furinghetti, F., & Morselli, F. (2009). Every unsuccessful problem solver is unsuccessful in his or her own way: Affective and cognitive factors in proving. *Educational Studies in Mathematics*, 70(1), 71–90.  
<https://doi.org/10.1007/s10649-008-9134-4>
- Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics*, 44(1–2), 5–23.  
<https://doi.org/10.1023/A:1012737223465>

- Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics*, 24(4), 389–399. <https://doi.org/10.1007/BF01273372>
- Kusnandi. (2008). *Pembelajaran matematika dengan strategi abduktif-deduktif untuk menumbuhkembangkan kemampuan membuktikan pada mahasiswa* [Doctoral dissertation, Universitas Pendidikan Indonesia].
- Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). Mathematical proof: The learning obstacles of pre-service mathematics teachers on transformation geometry. *Journal on Mathematics Education*, 10(1), 117–126. <https://doi.org/10.22342/jme.10.1.5379.117-126>
- Parsons, S., Croft, T., & Harrison, M. (2009). Does students' confidence in their ability in mathematics matter? *Teaching Mathematics and Its Applications*, 28(2), 53–68. <https://doi.org/10.1093/teamat/hrp010>
- Selden, A., & Selden, J. (2008). Overcoming students' difficulties in learning to understand and construct proofs. In *Making the connection* (pp. 95–110). Mathematical Association of America. <https://doi.org/10.5948/UPO9780883859759.009>
- Selden, A., & Selden, J. (2013). The roles of behavioral schemas, persistence, and self-efficacy in proof construction. *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education*, 246–255.
- Selden, J., Benkhalti, A., & Selden, A. (2014). An analysis of transition-to-proof course students' proof constructions with a view towards course redesign. *Proceedings of the 17th Annual Conference on Research in Undergraduate Mathematics Education*, 246–259.
- Shimizu, Y. (2022). Relation between mathematical proof problem solving, math anxiety, self-efficacy, learning engagement, and backward reasoning. *Journal of Education and Learning*, 11(6), 62. <https://doi.org/10.5539/jel.v11n6p62>
- Shongwe, B. (2025). A student's espoused beliefs about functions of proof and their behaviour in proving. *International Journal of Mathematical Education in Science and Technology*, 1–23. <https://doi.org/10.1080/0020739X.2025.2483507>
- Stylianou, D. A., Blanton, M. L., & Rotou, O. (2015). Undergraduate students' understanding of proof: Relationships between proof conceptions, beliefs, and classroom experiences with learning proof. *International Journal of Research in Undergraduate Mathematics Education*, 1(1), 91–134. <https://doi.org/10.1007/s40753-015-0003-0>
- Tall, D. (1998). The cognitive development of proof: Is mathematical proof for all or for some? *Conference of the University of Chicago School Mathematics Project*, 117–136.

- Viholainen, A., Tossavainen, T., Viitala, H., & Johansson, M. (2019). University mathematics students' self-efficacy beliefs about proof and proving. *LUMAT: International Journal of Math, Science and Technology Education*, 7(1).  
<https://doi.org/10.31129/LUMAT.7.1.406>
- Viteri, S., & DeDeo, S. (2022). Epistemic phase transitions in mathematical proofs. *Cognition*, 225, Article 105120.  
<https://doi.org/10.1016/j.cognition.2022.105120>
- Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*, 48(1), 101–119.  
<https://doi.org/10.1023/A:1015535614355>
- Weber, K. (2010). Mathematics majors' perceptions of conviction, validity, and proof. *Mathematical Thinking and Learning*, 12(4), 306–336.  
<https://doi.org/10.1080/10986065.2010.495468>