

## EXPLORING STUDENT'S MATHEMATICAL REASONING IN SOLVING PROBABILITY PROBLEMS BASED ON POLYA'S THEORY

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**ABSTRACT** Mathematical reasoning is a fundamental competency in mathematics learning, particularly in solving probability word problems that require systematic and logical thinking. One widely used framework to support problem solving is Polya's problem-solving theory, which consists of four stages: understanding the problem, devising a plan, carrying out the plan, and reviewing the solution. This study aims to analyze students' mathematical reasoning abilities in solving probability word problems based on Polya's steps. Polya's theory is employed as an analytical framework to examine how students reason through probability tasks. The study adopted a qualitative descriptive approach involving tenth-grade students at a vocational high school in Jambi City. Data were collected through probability word problem tests, interviews, and classroom observations. The findings indicate that students' mathematical reasoning abilities vary across the stages of Polya's problem-solving process. At the understanding stage, most students were able to identify relevant information, although some experienced difficulties with probability terminology. During the planning stage, only a small number of students could formulate systematic solution strategies. In the execution stage, many students performed basic calculations correctly, but errors emerged when applying formulas to more complex problems. The reviewing stage was the least practiced, resulting in many undetected errors. These results suggest that while students possess basic problem-solving skills, they require further guidance to develop systematic and reflective reasoning. Therefore, reinforcing each stage of Polya's problem-solving process is essential for improving students' mathematical reasoning in solving probability word problems.

**Keywords:** mathematical reasoning, probability word problems, Polya's problem-solving steps, qualitative analysis

**ABSTRAK** Penalaran matematis merupakan salah satu kompetensi dasar dalam pembelajaran matematika, khususnya dalam menyelesaikan soal cerita peluang yang menuntut kemampuan berpikir sistematis dan logis. Salah satu kerangka yang umum digunakan untuk mendukung pemecahan masalah adalah teori pemecahan masalah Polya

yang terdiri atas empat tahap, yaitu memahami masalah, merencanakan penyelesaian, melaksanakan rencana, dan memeriksa kembali hasil penyelesaian. Penelitian ini bertujuan untuk menganalisis kemampuan penalaran matematis peserta didik dalam menyelesaikan soal cerita peluang berdasarkan langkah-langkah Polya. Teori Polya digunakan sebagai kerangka analisis untuk mengkaji proses penalaran peserta didik dalam menyelesaikan permasalahan peluang. Penelitian ini menggunakan pendekatan deskriptif kualitatif dengan subjek peserta didik kelas X di salah satu SMK di Kota Jambi. Data dikumpulkan melalui tes soal cerita peluang, wawancara, dan observasi pembelajaran. Hasil penelitian menunjukkan bahwa kemampuan penalaran matematis peserta didik bervariasi pada setiap tahap pemecahan masalah menurut Polya. Pada tahap memahami masalah, sebagian besar peserta didik mampu mengidentifikasi informasi penting, meskipun masih terdapat kesulitan dalam memahami istilah peluang. Pada tahap perencanaan, hanya sedikit peserta didik yang mampu menyusun strategi penyelesaian secara sistematis. Pada tahap pelaksanaan, banyak peserta didik dapat melakukan perhitungan dasar dengan benar, namun masih terjadi kesalahan dalam penerapan rumus pada soal yang lebih kompleks. Tahap pemeriksaan kembali merupakan tahap yang paling jarang dilakukan sehingga banyak kesalahan tidak terdeteksi. Temuan ini menunjukkan bahwa meskipun peserta didik memiliki kemampuan dasar dalam menyelesaikan soal peluang, mereka masih memerlukan pembiasaan berpikir sistematis dan reflektif. Oleh karena itu, penguatan pada setiap tahap pemecahan masalah Polya menjadi penting untuk meningkatkan kemampuan penalaran matematis peserta didik dalam menyelesaikan soal cerita peluang.

**Kata-kata kunci:** penalaran matematis, soal cerita peluang, langkah Polya, analisis kualitatif

## INTRODUCTION

Mathematical reasoning ability is a fundamental aspect in mathematics learning that plays a crucial role in developing students' critical thinking and problem-solving skills, especially in probability material, which is both contextual and abstract (Ade Abimanyu & Pratama, 2023; Irianti, 2020; Jumiansih et al., 2020; Mutiah et al., 2023). Word problems in probability require students not only to memorize concepts but also to apply systematic thinking strategies in order to understand the problem, plan a solution, perform calculations, and verify the results. A widely used approach to guide students through this process is the problem-solving theory proposed by George Polya, which consists of four main stages: understanding the problem, planning the solution, carrying out the plan, and checking the results (Berutu & Juliani, 2024; Irianti, 2020).

Various studies have confirmed the effectiveness of using Polya's steps in improving students' mathematical reasoning ability in probability material (Fahrudin et al., 2019). For example, a study at MTs Muhammadiyah 1 Malang showed that students with high reasoning ability were able to carry out all stages of Polya effectively, while students with lower ability faced difficulties, particularly in the stages of understanding the problem and planning the solution (Irianti, 2020). Another study on prospective mathematics teachers at Satya Wacana Christian University found that linguistic and logical-mathematical intelligence influenced students' ability to fulfill Polya's stages, particularly in understanding the problem and checking the solution (Ade Abimanyu & Pratama, 2023). In addition, a study at MTs Darul Falah

Terpadu showed a difference in students' mathematical problem-solving abilities based on gender, using Polya's indicators (Mutiah et al., 2023).

The implementation of Polya's theory not only helps students solve probability problems accurately but also develops critical, creative, and systematic thinking skills, which are essential in mathematics learning in general (Berutu & Juliani, 2024; Chacón-Castro et al., 2023; Chusna & Rosyada, 2024; Suhartatik et al., 2023). Thus, analyzing students' mathematical reasoning abilities based on Polya's steps becomes important to identify the challenges and potential of students in solving probability word problems, which can serve as the foundation for developing more effective and student-centered learning strategies (Basir, Nur et al., 2022; Riyadi et al., 2021).

Previous studies have also highlighted the importance of a deep understanding of probability concepts and students' ability to connect these concepts with the context of word problems, which is part of the stage of understanding the problem in Polya's theory (Awuah & Ogbonnaya, 2020; Endrawati & Ramlah, 2021; Fitriana & Mampouw, 2019; Irianti, 2020). Students' difficulties in formulating a solution plan and verifying the results have also been a major focus in the literature related to mathematical problem-solving (Aini & Mukhlis, 2020; Muslimin & Sunardi, 2019; Mutiah et al., 2023; Purcar et al., 2024; Wulandari et al., 2024). Therefore, this article aims to comprehensively analyze students' mathematical reasoning abilities in solving probability word problems based on Polya's theory, with the hope of providing a clear picture of the problem-solving stages mastered by students and the challenges they face.

## METHODS

This study uses a qualitative descriptive method aimed at analyzing the mathematical reasoning abilities of grade X students at a vocational school in Jambi City in solving probability word problems based on Polya's steps. The qualitative descriptive method was chosen because it allows the researcher to deeply describe and analyze the students' thinking processes when facing mathematical problems, particularly those related to probability. The research subjects were purposively selected, meaning that grade X students who had already studied probability material were chosen, ensuring that they had the relevant knowledge base to solve the problems presented. The selection of subjects aims to focus the study on a group of students who have direct experience with the material being investigated.

The main instrument used in this study is a probability word problem test designed according to mathematical reasoning indicators and Polya's steps. This test is intended to assess the students' ability to apply Polya's steps in solving probability word problems, specifically the stages of understanding the problem, planning the solution, executing the plan, and checking the results. In addition to the test, in-depth interviews were used as an additional instrument to further explore the

students' thinking processes while solving the problems. The purpose of these interviews is to understand the reasoning behind each step taken by the students in solving the problem and to assess how well they apply Polya's theory in their problem-solving process.

Data were collected through three main methods: written tests, interviews, and observations. The written tests were given to measure students' abilities in solving probability word problems based on Polya's steps. In-depth interviews were conducted to further explore the students' thoughts and strategies in solving the problems. Observations were made to see directly how students interact with the problems and how they apply Polya's steps. The data collection process was carried out in March 2025, with the aim of obtaining valid and representative data regarding the students' mathematical reasoning abilities.

Once the data were collected, analysis was conducted by identifying and categorizing students' abilities at each stage of Polya's theory. This analysis aims to assess how well students master each stage in Polya's theory and to identify errors or challenges encountered by students at each stage. The validity of the data was ensured by using source and method triangulation. Source triangulation was carried out by comparing data obtained from the tests, interviews, and observations. Meanwhile, method triangulation was conducted by combining various data collection techniques to ensure consistency and reliability of the research findings. Additionally, to further enhance the validity of the data, discussions with mathematics education experts were held to ensure that the data interpretation aligns with existing theories and practices in mathematics education.

## **FINDING AND DISCUSSION**

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The research findings show that students' mathematical reasoning abilities in solving probability word problems based on Polya's steps vary. In the stage of understanding the problem, most students are able to identify key information and what is being asked in the problem, although some still experience confusion in understanding probability terms. In the planning stage, only about half of the students can formulate a systematic problem-solving strategy according to probability concepts, while the others tend to perform calculations without a clear plan. In the execution stage, most students can correctly perform basic probability calculations, but errors occur when applying formulas to more complex word problems. The checking stage is the least frequently performed by students; only a small portion review their answers and work processes, resulting in errors that could have been corrected. These findings suggest that, although students possess basic abilities in solving probability problems, they still need to practice systematic and reflective thinking according to Polya's stages to improve the quality of their mathematical reasoning.

## Results of Subject 1's Work

This probability word problem involves randomly selecting two students from a class of 20 students, consisting of 12 female students and 8 male students. The goal is to determine the probability that both selected students are male. The first step in solving this problem is to understand the problem, where Subject 1 identifies the relevant information, namely the number of male students (8) and the total number of students (20). Subject 1 understands that what is being asked is the probability of selecting two male students from the 8 male students available.

Jawab:

Diket: 12 Siswa Perempuan  
8 Siswa Laki-Laki  
 $n(S) = 20$  Siswa

Dit: Peluang bahwa kedua siswa yg dipilih adalah laki-laki

Jawab:  $C_r^n = \frac{n!}{(n-r)!r!}$

$$C_8^{20} = \frac{20!}{(20-8)!8!}$$

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{12! \cdot 8!}$$

$$= 164$$

$P(A) = \frac{n(A)}{n(S)}$

$$P(164) = \frac{164}{20}$$

$$= 8.2$$

Figure 1. The work results of Subject 1.

In the planning stage, Subject 1 chooses to use the combination formula to calculate the number of ways to select two students from a total of 20 students, as well as the number of ways to select two male students from the 8 male students. Subject 1 writes the combination formula to calculate the number of ways to choose two students from the total of 20 students using  $C(20,8)$ , which represents the number of combinations for selecting 8 students from the 20 available. After calculating the combination, Subject 1 proceeds to the probability calculation stage. The probability  $P(A)$  is calculated by comparing the number of ways to select two male students from 8 male students with the number of ways to select two students from the total of 20 students. In the execution stage, Subject 1 carefully performs the calculations to ensure the correct result. In the final stage, checking the result, Subject 1 evaluates whether the calculation result matches what is asked in the problem and ensures that there are no errors in the steps taken.

Polya's theory provides a systematic approach to solving this problem. In the first stage, understanding the problem, Subject 1 is able to identify the key elements in the problem, such as the total number of students and the number of male students.

In the second stage, planning the solution, Subject 1 selects the appropriate formula to calculate the combinations, both for selecting two students from the total of 20 students and for selecting two male students from the 8 male students. In the third stage, executing the plan, Subject 1 performs the calculations carefully, while in the final stage, checking the results, Subject 1 ensures that the obtained result aligns with what is asked in the problem.

Polya's theory is highly relevant to the mathematical reasoning indicators used in the process of solving this problem. In the understanding stage, Subject 1 demonstrates the ability to recognize and comprehend the relevant information in the problem, such as the number of male students and the total number of students. This indicator encompasses the ability to identify the elements of the problem that affect the probability calculation. In the planning stage, Subject 1 is able to select the correct formula to solve the problem, namely the combination formula for calculating the number of ways to choose students from two different groups. Subject 1 also shows their ability to plan the solution steps by choosing the formula that aligns with the context of the problem, which matches the mathematical reasoning indicator that requires students to select the appropriate method. In the execution stage, Subject 1 demonstrates skill in correctly applying the formula and calculating the appropriate values, reflecting the indicator of being able to compute the desired value or result using the correct method. Additionally, Subject 1 also demonstrates the ability to use mathematical representations accurately, namely by applying combinations to solve this problem.

In the final stage of checking the results, Subject 1 is expected to evaluate the calculation results and check their consistency with the given problem. At this stage, Subject 1 re-evaluates the calculations performed and ensures that there are no mistakes in the steps taken, in accordance with the indicator of checking results to ensure accuracy and consistency with the given problem. Although Subject 1 successfully solves this problem correctly, the findings suggest that some students still require further practice in systematic and reflective thinking habits, particularly at the stage of checking the results. Therefore, reinforcement at each stage of Polya's process is crucial for enhancing Subject 1's mathematical reasoning quality in solving probability word problems and understanding other mathematical concepts. In an interview with Subject 1, when asked why they chose the combination formula to solve this problem, Subject 1 responded that the formula was used because the students were selected randomly, and there was no specific order in choosing the students. This shows that Subject 1 understands that in this problem, the selection of two students is done randomly from the entire class, and thus the combination formula is the correct method for calculating the probability.

### **Results of Subject 2's Work**

This probability word problem involves randomly selecting two students from a class of 20 students, consisting of 12 female students and 8 male students. The goal is to

determine the probability that both selected students are male. In the understanding stage (mathematical reasoning indicator: Understanding the information given in the problem), Subject 2 successfully identifies the key information, namely the number of male students (8) and the total number of students (20). Subject 2 understands that the problem asks for the probability of selecting two male students from the 8 available male students. This shows that Subject 2 understands the problem and can identify the relevant elements for calculating probability.

Handwritten work by Subject 2:

Jawab : diket : terdapat 20 siswa  
 12 Siswa Perempuan  
 8 Siswa laki-laki.  
 2 Siswa yg dipilih mengikuti lomba

$$\text{Jawab : } C_r^n = \frac{n!}{(n-r)!r!}$$

$$C_2^{20} = \frac{20!}{(20-2)!2!}$$

$$= \frac{20!}{18! \cdot 2!}$$

$$= \frac{20 \cdot 19 \cdot \cancel{18!}}{\cancel{18!} \cdot 2}$$

$$= \frac{380}{2} = 190$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{190}{64} = 2,96$$

Figure 2. The work results of Subject 2.

In the planning stage (mathematical reasoning indicator: Ability to choose the appropriate method), Subject 2 selects the correct combination formula to calculate the number of ways to choose two students from the total of 20 students. Using the combination formula, Subject 2 correctly calculates the number of ways to choose two students from the total of 20 students, which is  $C(20,2) = 190$ . Next, Subject 2 calculates the combination  $C(8,2)$  to choose two male students from the 8 male students and obtains the result of 28. At this stage, Subject 2 has demonstrated the ability to select the appropriate method and plan the problem-solving steps.

In the execution stage (mathematical reasoning indicator: Ability to correctly and precisely apply formulas), Subject 2 continues with the calculation and successfully computes the combination value correctly. However, there is an error in the application of the probability formula. Subject 2 calculates the probability using the classical formula  $P(A) = n(A) / n(S)$ , but the result indicates a value greater than 1 (2.96), which is impossible for a probability. This reveals an error in applying the probability formula. Although Subject 2 correctly applies the formula for calculating the combinations, the mistake occurs in the application of the probability formula. At the checking results stage (mathematical reasoning indicator: Checking results

and ensuring their consistency with the problem), Subject 2 should have reviewed the obtained result to ensure that the calculated probability makes sense (the probability value should not exceed 1). However, since Subject 2 did not sufficiently check the results, the error in the probability calculation remained undetected. Therefore, even though the checking stage was performed, the inconsistency in the results shows that this step was not fully carried out with the necessary care.

In an interview with Subject 2, when asked why they chose the combination formula to solve this problem, Subject 2 responded that the formula was used because the problem asks for the probability of selecting male students. This shows that Subject 2 understands the context of the problem, which focuses on selecting male students, and therefore, the combination formula is the correct choice for calculating the probability. Overall, although Subject 2 followed the correct steps in using the combination formula, the error in applying the probability formula indicates a lack of precision and understanding of probability. Using Polya's Theory, the first stage of understanding the problem and the second stage of planning the solution were done well. However, in the third stage of executing the plan and the fourth stage of checking the results, Subject 2 encountered difficulties. This highlights the importance of reinforcement in the final stages to ensure that the calculations are more accurate and consistent.

### Results of Subject 3's Work

$r = (2)$

jawab: banyak siswa = 20 siswa ( $n$ )  
 Diketahui:  $P(A)$  / siswa perempuan = 12 siswa  
 $P(B)$  / siswa laki-laki = 8 siswa  
 Ditanya: tentukan peluang bahwa ke 2 siswa yang dipilih untuk mengikuti lomba adalah laki-laki?

jawab  
 kombinasi:  $C_r^n = \frac{n!}{(n-r)! r!}$  ~~Salah~~  
 $= \frac{20!}{(20-2)! 2!}$   
 $= \frac{20!}{18! 2!}$   
 $= \frac{20 \cdot 19 \cdot 18!}{18! \cdot 2 \cdot 1} = \frac{20 \cdot 19}{2} = \frac{380}{2} = 190$

Figure 3. The work results of Subject 3.

This probability word problem involves randomly selecting two students from a class of 20 students, consisting of 12 female students and 8 male students. The goal is to determine the probability that both selected students are male. In the understanding stage (mathematical reasoning indicator: Understanding the

information given in the problem), Subject 3 identifies the key information, namely the number of male students (8) and the total number of students (20). Subject 3 understands that the problem asks for the probability of selecting two male students from the 8 available male students, demonstrating a basic understanding of the problem context. In the planning stage (mathematical reasoning indicator: Ability to choose the appropriate method), Subject 3 selects the combination formula to calculate the number of ways to choose two students from the total of 20 students and the number of ways to choose two male students from the 8 male students. Subject 3 writes the combination formula to calculate the number of ways to choose two students from the total of 20 students using  $C(20,2)$ , but an error occurs in the calculation and application of this formula. Subject 3 calculates  $C(20,2)$ , but the subsequent steps are not in accordance with the correct process, causing the calculation to be inaccurate. Subject 3 needs to correct this calculation step to ensure the correct result.

In the execution stage (mathematical reasoning indicator: Ability to correctly and precisely apply formulas), Subject 3 continues the calculation using the combination formula. However, the calculation results in a number larger than expected, 380, which should represent the number of ways to choose two students from the total of 20 students. This result, however, does not align with the expected outcome for the probability requested. The error occurs in the application of the formula and the incorrect application of the probability calculation.

In the checking results stage (mathematical reasoning indicator: Checking results and ensuring their consistency with the problem), Subject 3 did not sufficiently review the calculations performed. The result obtained shows an unrealistic probability value, greater than 1, indicating an error in the previous steps. Therefore, it is important for Subject 3 to be more careful in reviewing the calculation results, ensuring that the results are consistent with the problem context and do not exceed the maximum possible value in probability calculations. In an interview with Subject 3, when asked why they chose the combination formula to solve this problem, Subject 3 responded that they did not understand the probability formula and only used the combination formula to calculate the number of ways to select two male students because they were more familiar with the combination formula. This shows that Subject 3 did not fully understand the concept of probability and preferred to use a formula that was easier for them to understand, even though it did not yield an accurate result in the context of this problem. Overall, although Subject 3 chose the correct combination formula to calculate the number of ways to select students, the error in applying the probability formula indicates a lack of understanding of probability concepts. Strengthening the understanding of the probability formula and reviewing the results is crucial to ensure that the solution provided is accurate and consistent with the problem given.

Based on the analysis of student work and existing literature, the following conclusions can be drawn regarding mathematical reasoning and the errors that occurred during the process of solving probability problems, referring to Polya's theory. This study shows that students demonstrate varying levels of ability at each stage of Polya's steps. In the understanding stage, most students are able to identify relevant information and the question, although some struggle with interpreting the term "probability" (Hendricks & Olawale, 2023; Tanzimah & Dina Sutrianti, 2023). This is consistent with the findings of Setiani et al. (2024), which indicate that students often struggle to understand problems, particularly in more abstract topics like probability, which hampers their ability to solve problems systematically (Hadfield, 2021). Research by Säfström (2024) also supports this, noting that students' difficulties in understanding the problem are central challenges when they attempt to engage in problem-solving. In the planning stage, only half of the students are able to formulate a problem-solving strategy systematically, while the others proceed with calculations without a clear plan. This trend is also found in the literature, where students frequently have difficulty transforming the problem into a solvable mathematical model (Natalia & Mampouw, 2024; Tanzimah & Dina Sutrianti, 2023).

The research by Putri et al. (2024) also confirms that the use of a cycle-based learning approach can improve students' understanding in planning mathematical problem-solving steps in a more structured manner. This finding aligns with the work of Romadhon et al. (2024), who discuss the importance of structured methods, such as Problem-Based Learning (PBL), in helping students organize and plan their approaches more effectively. In the execution stage, most students perform basic calculations correctly; however, errors occur in applying the formulas to more complex problems, reflecting students' difficulty in transforming word problems into the correct mathematical form (Tanzimah & Dina Sutrianti, 2023). Finally, in the checking results stage, students rarely review their answers, leading to missed opportunities to identify and correct mistakes (Tanzimah & Dina Sutrianti, 2023). This aligns with findings from other studies that show students often make errors due to rushing and not reviewing their steps (Putri et al., 2024).

Thus, Polya's theory remains a valuable framework for guiding students through structured problem-solving, although consistent practice and teaching are necessary to minimize errors, particularly in understanding the problem and applying mathematical operations.

## **CONCLUSIONS AND RECOMMENDATIONS**

Based on the research findings, it can be concluded that students' mathematical reasoning abilities in solving probability word problems based on Polya's theory show variability. In the understanding stage, most students can identify relevant information, although some still struggle with understanding terms related to probability. In the planning stage, many students are unable to formulate a solution strategy systematically, which affects the accuracy of the subsequent steps taken. In

the execution stage, while most students can perform basic calculations correctly, errors still occur in applying formulas to more complex word problems. The checking results stage is the most frequently overlooked, resulting in many errors that could have been corrected. These findings align with previous research, which shows that students' errors in solving mathematical problems, including probability problems, are largely influenced by a lack of deep understanding of the proper problem-solving procedures, as well as errors in understanding problem information, transforming information into the appropriate mathematical form, and mistakes in writing the final answer. Therefore, it is important for students to receive further training in applying the systematic steps of Polya's theory, so they can improve the quality of their mathematical reasoning in solving probability word problems and correct the common errors that occur at each stage of problem-solving.

## REFERENCES

- Ade Abimanyu, S., & Pratama, F. W. (2023). Analisis pemecahan masalah matematika pada mahasiswa calon guru matematika dengan tipe kecerdasan linguistik dan logis-matematis. *Jurnal Ilmiah Pendidikan Citra Bakti*, 10(3), 673–683. <https://doi.org/10.38048/jipcb.v10i3.1716>
- Aini, N. N., & Mukhlis, M. (2020). Analisis kemampuan pemecahan masalah pada soal cerita matematika berdasarkan teori Polya ditinjau dari adversity quotient. *Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika*, 2(1), 105–128. <https://doi.org/10.35316/alifmatika.2020.v2i1.105-128>
- Awuah, F. K., & Ogbonnaya, U. I. (2020). Grade 12 students' proficiency in solving probability problems involving contingency tables and tree diagrams. *International Journal of Instruction*, 13(2), 819–834. <https://doi.org/10.29333/iji.2020.13255a>
- Basir, W. N., Kristiawati, & Usman, M. R. (2022). Analisis kemampuan penalaran matematis siswa ditinjau dari gaya kognitif. *Jurnal MathEdu (Mathematic Education Journal)*, 5(3), 17–26.
- Berutu, N. A., & Juliani, S. F. (2024). Implementasi teori Polya terhadap pemecahan masalah melalui materi bilangan cacah pada siswa sekolah dasar. *Jurnal Pendidikan Tambusai*, 8(1), 1753–1757.
- Chacón-Castro, M., Buele, J., López-Rueda, A. D., & Jadán-Guerrero, J. (2023). Pólya's methodology for strengthening problem-solving skills in differential equations: A case study in Colombia. *Computers*, 12(11), Article 239. <https://doi.org/10.3390/computers12110239>
- Chusna, A., & Rosyada. (2024). Analysis of students' mathematical reasoning in solving story problems according to Polya's steps. *Riemann: Research of*

*Mathematics and Mathematics Education*, 6(3), 274–286.  
<https://doi.org/10.38114/riemann.v6i3.61>

- Endrawati, P., & Ramlah. (2021). Analisis kemampuan penalaran matematis pada materi peluang ditinjau dari kemampuan awal siswa. *MAJU: Jurnal Ilmiah Pendidikan Matematika*, 8(2), 148–158.
- Fahrudin, D., Mardiyana, & Pramudya, I. (2019). The analysis of mathematic problem-solving ability by Polya steps on trigonometric material reviewed from self-regulated learning. *Journal of Physics: Conference Series*, 1254(1), Article 012076. <https://doi.org/10.1088/1742-6596/1254/1/012076>
- Fitriana, I. N., & Mampouw, H. L. (2019). Skema kognitif siswa dalam menyelesaikan soal peluang ditinjau dari pendekatan Polya. *Mosharafa: Jurnal Pendidikan Matematika*, 8(3), 353–364. <https://doi.org/10.31980/mosharafa.v8i3.572>
- Hadfield, K. L. F. (2021). Providing ability to probability: Reducing cognitive load through worked-out examples. *Teaching Statistics*, 43(1), 28–35. <https://doi.org/10.1111/test.12244>
- Hendricks, W., & Olawale, B. E. (2023). Mathematical probability: Learners' misconceptions in a selected South African school. *Infinity Journal*, 12(1), 165–178. <https://doi.org/10.22460/infinity.v12i1.p165-178>
- Irianti, N. P. (2020). Analisis kemampuan penalaran siswa dalam memecahkan masalah matematika berdasarkan langkah-langkah Polya. *MUST: Journal of Mathematics Education, Science and Technology*, 5(1), 80–94. <https://doi.org/10.30651/must.v5i1.3622>
- Jumiarsih, D. I., Kusmayadi, T. A., & Fitriana, L. (2020). Students' mathematical reasoning ability viewed from self-efficacy. *Journal of Physics: Conference Series*, 1538(1), Article 012101. <https://doi.org/10.1088/1742-6596/1538/1/012101>
- Muslimin, M., & Sunardi, S. (2019). Analisis kemampuan penalaran matematika siswa SMA pada materi geometri ruang. *Kreano: Jurnal Matematika Kreatif-Inovatif*, 10(2), 171–178. <https://doi.org/10.15294/kreano.v10i2.18323>
- Mutiah, S., Rohman, N., & Hasanudin, C. (2023). Analisis kemampuan pemecahan masalah matematis siswa kelas VIII MTs Darul Falah Terpadu pada materi peluang ditinjau dari gender. *Jurnal Edumatic: Jurnal Pendidikan Matematika*, 4(2), 7–17. <https://doi.org/10.21137/edumatic.v4i2.789>
- Natalia, A., & Mampouw, H. (2024). Analisis kesalahan siswa dalam menyelesaikan soal peluang berdasarkan teori Newman ditinjau dari gaya belajar. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 8(1), 12–25.
- Purcar, A. M., Bocoş, M., Pop, A. L., Roman, A., Rad, D., Mara, D., ... Triff, D. G. (2024). The effect of visual reasoning on arithmetic word problem solving. *Education Sciences*, 14(3), Article 278. <https://doi.org/10.3390/educsci14030278>

- Putri, D. U., Sirwanti, & Aspikal. (2024). Improving students' mathematical reasoning through the learning cycle instructional model. *Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika*, 7(2), 326–340. <https://doi.org/10.30606/absis.v7i2.2979>
- Riyadi, S., T., & Nikmaturrohmah, P. (2021). Profile of students' problem-solving skills viewed from Polya's four-steps approach in elementary school students. *European Journal of Educational Research*, 11(2), 859–872.
- Romadhon, N. I., Amir, M. F., & Wardana, M. D. K. (2024). Assessing students' mathematical reasoning in problem-based learning: A gender perspective. *International Journal of Evaluation and Research in Education*, 13(6), 3763–3774. <https://doi.org/10.11591/ijere.v13i6.29580>
- Säfström, A. I. (2024). Developing a diagnostic framework for primary and secondary students' reasoning difficulties during mathematical problem solving. *Educational Studies in Mathematics*, 115(2), 125–149. <https://doi.org/10.1007/s10649-023-10278-1>
- Setiani, N., Saragih, S., & Yuanita, P. (2024). Pengembangan modul berbasis pendekatan heuristik untuk meningkatkan kemampuan pemecahan masalah matematis siswa pada materi peluang kelas VIII SMP. *Jurnal Absis: Jurnal Pendidikan Matematika dan Matematika*, 6(2), 976–987. <https://doi.org/10.30606/absis.v6i2.2338>
- Suhartatik, P., Susiswo, S., & As'ari, A. R. (2023). Penalaran matematis siswa dalam menyelesaikan masalah pola bilangan dan scaffolding-nya. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 7(1), 432–441. <https://doi.org/10.31004/cendekia.v7i1.1068>
- Tanzimah, T., & Sutrianti, D. (2023). Analisis kesalahan peserta didik dalam menyelesaikan soal cerita pada materi peluang berdasarkan Newman's error analysis (NEA). *Indiktika: Jurnal Inovasi Pendidikan Matematika*, 5(2), 191–200. <https://doi.org/10.31851/indiktika.v5i2.11469>
- Wulandari, R., Haryono, Y., & Lovia, L. (2024). Analisis kemampuan pemecahan masalah dalam menyelesaikan soal matematika di SMK. *Jurnal Matematika dan Ilmu Pengetahuan Alam*, 2(1), 116–122. <https://doi.org/10.59581/konstanta-widyakarya.v2i1.2092>