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ANALYSIS OF STUDENTS' MATHEMATICAL REPRESENTATION BASED ON LEVELS OF MATHEMATICAL ABILITY

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ABSTRACT Mathematical representation is a fundamental skill in mathematics learning, allowing students to express and solve problems through symbolic, verbal, and visual forms. However, students' varying levels of mathematical ability can significantly influence the quality of their representational skills. This study employed a descriptive qualitative method involving three eighth-grade students from SMPN 2 Kasimbar, selected based on their mathematical abilities: high, medium, and low. Data were collected through tests and semi-structured interviews, and analyzed using data condensation, data display, and conclusion drawing techniques. The results indicated that high-ability students could construct and solve systems of linear equations in two variables, although they exhibited some difficulty in interpreting variables accurately. Medium-ability students demonstrated proficiency in formulating equations and performing calculations, with particular strength in verbal and visual representations. In contrast, low-ability students struggled with algebraic operations, often misused variables, and were unable to generate appropriate graphical representations. These findings highlight the significant role of mathematical ability in shaping students' representation skills and underscore the need for teachers to implement more adaptive and differentiated instructional strategies.

Keywords: mathematical representation, mathematical ability, system of linear equations in two variables, qualitative approach

ABSTRAK Representasi matematis merupakan keterampilan mendasar dalam pembelajaran matematika yang memungkinkan siswa mengekspresikan dan menyelesaikan masalah melalui bentuk simbolik, verbal, dan visual. Namun, perbedaan tingkat kemampuan matematis siswa dapat memengaruhi kualitas keterampilan representasi tersebut. Penelitian ini menggunakan metode deskriptif kualitatif dengan melibatkan tiga siswa kelas VIII dari SMPN 2 Kasimbar yang dipilih berdasarkan tingkat kemampuan matematisnya: tinggi, sedang, dan rendah. Data dikumpulkan melalui tes dan wawancara semi-terstruktur, lalu dianalisis menggunakan teknik kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa siswa berkemampuan tinggi mampu menyusun dan menyelesaikan sistem persamaan linear dua variabel, meskipun masih mengalami kesulitan dalam menginterpretasikan variabel dengan tepat. Siswa berkemampuan sedang menunjukkan kecakapan dalam merumuskan persamaan dan melakukan perhitungan, serta unggul dalam representasi verbal dan visual. Sementara itu, siswa berkemampuan rendah mengalami kesulitan dalam operasi aljabar, sering salah menggunakan variabel, dan tidak mampu merepresentasikan masalah secara grafis. Temuan ini menunjukkan pentingnya peran tingkat kemampuan matematis dalam membentuk keterampilan representasi siswa dan menekankan perlunya strategi pembelajaran yang lebih adaptif dan terdiferensiasi.

Kata-kata kunci: representasi matematis, kemampuan matematis, sistem persamaan linear dua variabel, pendekatan kualitatif

INTRODUCTION

Mathematics plays a crucial role in various fields and in everyday life. Therefore, it is taught from elementary education through to higher education (Ario, 2019). One of the general goals of mathematics learning is the development of mathematical representation (Fadhilah et al., 2019). Representation refers to the expression of mathematical thinking or ideas that students use to find solutions to the problems they encounter (Azzahra & Sopiany, 2023). It plays a central role in enabling students to communicate, analyze, and make sense of mathematical situations, especially at the secondary education level.

In line with this, Duval (2006) emphasized that "the crucial point is that mathematical objects are not accessible by perception, and the only way to get access to them is through representations." However, he also pointed out that the use of representations introduces specific cognitive functions not required in other domains. Representation can be seen as a description of an object or process—through words, diagrams, or graphics (Sahendra et al., 2018). It is also viewed as a mental image a person uses to solve problems, which can be visualized in verbal, symbolic, or visual form (Yanti et al., 2018). Thus, mathematical representation helps students understand and express mathematical ideas through various forms such as symbols, words, and images. This ability serves as a bridge between abstract concepts and concrete forms, making them easier for students to comprehend (Azzahra & Sopiany, 2023).

The importance of this ability has been institutionalized through standards. The National Council of Teachers of Mathematics (NCTM) has established representation as one of the five key process standards in mathematics learning—alongside problem solving, reasoning and proof, communication, and connections (Siagian, 2016). Representation skills not only support conceptual understanding but are also highly relevant in solving problems such as systems of linear equations in two variables, which require modeling, interpretation, and visualization.

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Solving systems of linear equations in two variables necessitates the coordination of multiple representations—symbolic, graphical, and verbal—to understand and interpret mathematical relationships effectively. This multifaceted approach engages students in complex cognitive processes, fostering deeper comprehension and problem-solving abilities. As noted by Roberts and Le Roux (2019), students often rely on procedural methods without fully grasping the underlying concepts, highlighting the need for instructional strategies that emphasize the integration of various representations. Furthermore, the ability to transition between different forms of representation is crucial for developing a robust understanding of mathematical concepts (Minarni, Napitupulu, & Husein, 2016).

However, interviews with mathematics teachers of eighth-grade students at SMPN 2 Kasimbar revealed that despite instruction involving explanations and example problems, many students still struggle with questions that require mathematical representation. This is evident from the students' answers, which often contain significant errors. Teachers believe this is due to students' insufficient understanding of how to develop and express mathematical concepts effectively.

These difficulties highlight a gap between instructional expectations and classroom realities. One key factor contributing to this gap is the variation in students' mathematical abilities. According to Apriatni et al. (2022), mathematical ability is a necessary skill for solving mathematical problems. Widarti (2019) and Julaeha et al. (2020) categorize students' mathematical ability into three levels: high, medium, and low.

Although there have been several studies on mathematical representation, most have focused on general aspects or examined it through other lenses, such as gender differences (Umaroh & Pujiastuti, 2020). There is still a lack of studies that specifically analyze how different levels of mathematical ability influence students' forms of representation in solving systems of linear equations in two variables.

Based on this background, this study aims to analyze how students with different levels of mathematical ability demonstrate their representation skills when solving systems of linear equations in two variables.

METHODS

This research employs a descriptive qualitative approach, aiming to gain an in-depth understanding of students' mathematical representations based on their levels of mathematical ability. The study was conducted at SMP Negeri 2 Kasimbar, located in Posona Village, Kasimbar Sub-district, Parigi Moutong Regency, during the odd semester of the 2024/2025 academic year. The subjects of the study were three eighth-grade students (class VIII B), selected purposively based on their levels of mathematical ability: high, medium, and low. The categorization was based on their mathematics report card scores from the even semester of the 2023/2024 academic year. Grouping was carried out with reference to the mean score and standard deviation, following the guidelines by (Arikunto, 2012).

The instruments used in this research consisted of: 1) Primary instrument: The researcher (human instrument), acting as data collector, observer, and analyst. 2)Supporting instruments: A mathematical representation test and a semi-structured interview guide. (Sugiyono, 2013) The test was developed based on indicators of mathematical representation (mathematical expressions, written words/text, and visuals) relevant to the topic of systems of linear equations in two variables. Validation was conducted by a subject matter expert (content validator) through a review process to ensure alignment with the research objectives. Feedback from the validator was implemented in the form of revisions to question wording, appropriate terminology, and clarity of task instructions.

Data were collected through: 1) Written tests: Questions designed to assess students' mathematical representation abilities in symbolic, verbal, and visual aspects. 2) Semi-structured interviews: Conducted to further explore students' thought processes and reasoning in response to the test items. These two techniques were applied sequentially, beginning with the test and followed by interviews based on students' answers.

Data analysis followed the Miles and Huberman model, consisting of three stages: 1) Data Reduction: Sorting, simplifying, and focusing the data obtained from the test and interviews. Irrelevant data were discarded, while the main data were organized based on representation indicators. 2) Data Display: Presenting data in descriptive narrative form and summary tables to facilitate interpretation and pattern identification according to mathematical ability categories. 3) Conclusion Drawing and Verification: Conclusions were drawn from the patterns identified in the data and were verified through time triangulation (repetition of tests and interviews) to ensure the credibility of the findings.

FINDING AND DISCUSSION

This study explores how students of different mathematical ability levels represent their thinking when solving problems involving systems of linear equations in two variables. The focus is on three types of mathematical representation: symbolic (expressions/equations), verbal (written and oral communication), and visual (graphical representation on a Cartesian plane). These forms are analyzed based on the observed performance of three students representing high, medium, and low mathematical ability levels.

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Table 1. Students' Mathematical Representation Abilities Based on Ability Levels

Representation Type	High-Ability Student	Medium-Ability Student	Low-Ability Student
Symbolic	Constructed correctly, but variable assumptions inaccurate	Constructed with accurate structure and variables	Constructed correctly, but with incorrect variable assumptions
Verbal	Logical conclusion, written explanation, but used "group" as variable	Oral explanation and correct conclusion	Misinterpreted problem, no conclusion
Visual	Accurate graph, unlabeled intersection point	Accurate graph, unlabeled intersection point	Unable to draw graph

Symbolic Representation in Constructing Mathematical Models

Symbolic representation involves the translation of real-world problems into mathematical expressions, often through the assignment of variables and the formation of equations. In this study, all three students were able to construct systems of equations based on the information provided. However, the quality and accuracy of their symbolic representations varied significantly.

The medium-ability student demonstrated the most accurate symbolic model, correctly interpreting the context and assigning appropriate variables. This indicates a strong grasp of the *referential function* of symbols, as discussed by Duval (2006), where symbols are not only manipulated but understood in connection with their real-world referents.

In contrast, both the high- and low-ability students exhibited a common error—using categorical labels such as "group" rather than quantitative variables such as "number of students." This reflects a misconception in understanding variables as abstract placeholders for numerical values, a difficulty noted in prior research by Roberts and Le Roux (2019), who emphasize that students often lack semantic awareness when engaging with algebraic expressions.

While these students were procedurally able to form equations, their misunderstanding of variables suggests limited conceptual understanding, which is critical for meaningful algebraic reasoning (Kieran, 2007). This discrepancy between mechanical fluency and conceptual accuracy is a significant challenge in mathematics education, especially at the middle school level.

Verbal Interpretation and Mathematical Communication

Verbal representation refers to students' ability to express mathematical reasoning and conclusions through spoken or written language. It reflects their internal comprehension and their capacity to communicate understanding clearly and logically.

The high-ability student was able to complete the task and write a conclusion that aligned with the solution, although the use of non-quantitative variable definitions (e.g., "group") slightly hindered clarity. The medium-ability student presented a coherent verbal explanation orally, suggesting solid internal comprehension, though lacking written articulation.

The low-ability student, however, struggled to verbalize or write a valid conclusion, indicating a gap not only in understanding but also in reflection. This aligns with Goldin's (1998) theory of internal representations, which are the mental structures that underpin mathematical reasoning. According to Minarni, Napitupulu, and Husein (2016), students with weak internal representations often fail to articulate or validate their solutions, showing surface-level engagement with the material.

Moreover, the absence of written conclusions by two out of three students points to a lack of *reflective habits*—an important aspect of mathematical communication and metacognition. Such habits can be nurtured through instructional strategies that emphasize *thinking aloud*, journaling, and collaborative discussions.

Visual Modeling and Graphical Translation

Visual representation involves the translation of algebraic expressions into graphical models, typically on the Cartesian coordinate plane. This process requires not only understanding the relationship between variables but also the spatial ability to interpret and construct graphs.

Both the high- and medium-ability students were able to produce accurate graphs, identifying coordinate pairs from equations and plotting them correctly. However, they failed to label the point of intersection, which is a crucial part of interpreting solutions to systems of equations. This oversight suggests a procedural understanding of graphing but an underdeveloped sense of mathematical communication and completeness.

The low-ability student was unable to draw the graph, primarily due to difficulty in converting algebraic equations into coordinate form. This finding aligns with Mainali (2021), who emphasizes that low-performing students often experience disconnections between algebraic reasoning and spatial representation. Janvier (1987) also points out that the ability to shift among representation modes—such as from symbolic to visual—is a hallmark of deep mathematical understanding.

This suggests that instruction should include explicit teaching on how different representations connect and how to document solutions clearly—not only for assessment purposes but to reinforce comprehension.

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Taken together, the findings reveal a pattern of differentiated strengths and challenges across student ability levels. The high-ability student displayed procedural fluency but exhibited conceptual gaps in defining variables and fully documenting conclusions. The medium-ability student showed balanced performance across forms of representation, albeit with limited written reflection. The low-ability student encountered substantial difficulties across all areas, particularly in verbal and visual representations.

These results support the view that students' mathematical ability levels influence the quality and completeness of their representations. Instruction that prioritizes procedural skills alone—without nurturing conceptual understanding and mathematical communication—may leave significant learning gaps. This highlights the importance of integrative teaching approaches that address multiple modes of representation and offer scaffolding tailored to students' ability levels (Roberts & Le Roux, 2019; Minarni et al., 2016; Janvier, 1987).

CONCLUSIONS AND RECOMMENDATIONS

This study highlights distinct differences in students' mathematical representation abilities—symbolic, verbal, and visual—based on their levels of mathematical proficiency. High-ability students were able to construct and solve systems of linear equations in two variables procedurally, and their graphs were accurate. However, they showed conceptual weaknesses in interpreting variables, often treating them as categorical entities rather than numerical quantities, which affected the clarity of their verbal explanations.

Medium-ability students demonstrated balanced performance across all representation types. They accurately formulated equations, arrived at correct solutions, and conveyed their reasoning coherently—though often orally rather than in writing. Their visual representations were also accurate, indicating a functional understanding of the connection between algebraic and graphical forms.

Low-ability students faced significant challenges, particularly in understanding variables and performing algebraic operations. While they could construct basic equations, these were often flawed, and they were unable to complete graphical representations. These difficulties indicate a lack of conceptual foundation and an inability to transition between forms of representation.

These findings underscore the importance of integrating conceptual development with procedural practice in mathematics instruction. Representation is not merely a skill of drawing or writing symbols—it reflects students' internal understanding of mathematical ideas and their ability to communicate them.

Teachers should place greater emphasis on explicitly teaching the meaning and role of variables, and on helping students connect symbolic, verbal, and visual representations. Providing structured scaffolding, encouraging reflective explanation (both written and oral), and integrating multiple representation tasks can enhance students' representational competence. Differentiated instruction based on students' ability levels may also be necessary to support those who require more targeted conceptual guidance.

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