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MAX-PLUS ALGEBRA MODELING FOR OUTPATIENT SERVICE QUEUES IN YOGYAKARTA PRIVATE HOSPITALS

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ABSTRACT This study aims to model the outpatient service queue system in a private hospital in Yogyakarta using the Max-Plus algebra approach and to represent the model using a Petri net. The research was motivated by the complexity and inefficiency observed in outpatient queues, which often lead to long and unpredictable waiting times. Data were collected through direct observation of the outpatient process on the fourth floor of a private hospital. The sequence of services was first illustrated using a flowchart to map each stage experienced by patients. The time data for each process were then used to construct a Max-Plus algebra matrix, providing a mathematical model of the system. This model was further expressed in the form of a Petri net to illustrate the discrete and sequential nature of the service flow. The simulation was performed using Scilab software to analyze the system's dynamics. Results from the simulation revealed that the service system is non-periodic, indicated by the absence of an eigenvalue. This suggests that the total service time may increase as the number of patients grows. The findings provide insight into the structure of the outpatient queue system and offer a mathematical framework for future system analysis or optimization.

Keywords: Max-Plus algebra, Petri net, outpatient queue, simulation, mathematical modeling

ABSTRAK Penelitian ini bertujuan untuk memodelkan sistem antrian layanan rawat jalan di rumah sakit swasta di Yogyakarta menggunakan pendekatan aljabar Max-Plus, serta merepresentasikan model tersebut dalam bentuk jaringan Petri. Penelitian ini dilatarbelakangi oleh kompleksitas dan ketidakefisienan dalam sistem antrian pasien rawat jalan, yang sering kali menyebabkan waktu tunggu yang panjang dan tidak terprediksi. Data diperoleh melalui observasi langsung terhadap proses layanan di lantai 4 salah satu rumah sakit swasta. Urutan pelayanan digambarkan terlebih dahulu dalam bentuk diagram alir berdasarkan tahapan-tahapan yang dialami pasien. Selanjutnya, data waktu dari setiap proses digunakan untuk membentuk model matematis berupa matriks Max-Plus. Model tersebut kemudian dinyatakan dalam bentuk jaringan Petri untuk menggambarkan sifat sistem layanan yang diskrit dan berurutan. Simulasi dilakukan menggunakan perangkat lunak Scilab untuk menganalisis dinamika sistem. Hasil simulasi menunjukkan bahwa sistem layanan bersifat non-periodik, yang ditunjukkan dengan tidak ditemukannya nilai eigen. Hal ini mengindikasikan bahwa waktu layanan total dapat meningkat seiring bertambahnya jumlah pasien. Temuan ini memberikan pemahaman tentang struktur sistem antrian rawat jalan dan dapat menjadi dasar untuk analisis dan evaluasi sistem lebih lanjut secara matematis.

Kata-kata kunci: aljabar Max-Plus, jaringan Petri, antrian rawat jalan, simulasi, pemodelan matematis

INTRODUCTION

Queueing systems are a significant subject of study in queueing theory and discrete system modeling, as they represent real-world situations where the demand for services exceeds the available service capacity. This phenomenon is commonly found in various sectors of everyday life, including transportation, banking, and healthcare. When service capacity fails to accommodate high demand, users are forced to wait, which can lead to several negative consequences. Long queues not only reduce efficiency and productivity but also harm the service provider's reputation and profitability (Hariputra, Defit, & Sumijan, 2022; Paulina Maure et al., 2021).

One of the most critical types of queueing systems can be found in healthcare facilities, particularly in hospitals. According to the Ministry of Health, hospitals are healthcare service facilities that provide comprehensive medical services, including outpatient, inpatient, and emergency care. In Yogyakarta, there is a private hospital known for its high-quality service, making it a primary referral center for a wide range of medical needs. The high volume of patients, especially during peak hours, often leads to long queues in outpatient services.

To address queue-related problems and analyze service systems systematically, mathematical modeling is essential. One effective approach is the use of Max-Plus algebra. Max-Plus algebra is a mathematical structure defined on the extended set of real numbers $\mathbb{R} \cup \{-\infty\}$, using the maximum operator ($\oplus$) for addition and regular addition ($\otimes$) for multiplication (Rudhito, 2016). This algebra has several applications in modeling dynamic systems, such as transportation scheduling, production systems, shortest path analysis (Subiono, 2015), and queueing networks (Rudhito, 2016). Employing Max-Plus algebra allows for efficient and structured modeling of discrete service system dynamics in a mathematical framework.

Max-Plus algebra is a mathematical structure defined over the set $\mathbb{R} \cup \{-\infty\}$, equipped with two operations: the maximum ($\oplus$) and the usual addition ($\otimes$). According to Rudhito (2016), for $\alpha \in \mathbb{R}$_max and matrices A, B $\in \mathbb{R}_{max}^{m \times n}$, the scalar-matrix product $\alpha \otimes A$ results in a matrix where each element is computed as $(\alpha \otimes A)_{ij} = \alpha \otimes A_{ij}$ for i = 1,…,m and j = 1,…,n. Meanwhile, the matrix product A $\otimes$ B is defined by $(A \otimes B\,)_{ij} = \oplus_{k=1}^{p} A_{ik} \otimes B_{kj}$ for the same indices.

Some properties of Max-Plus algebra, as stated in Theorem 2.2.4 (Rudhito, 2016), include:

Associativity: $(A \oplus B) \oplus C = A \oplus (B \oplus C)$,

Commutativity: $A \oplus B = B \oplus A$,

Distributivity: $A \otimes (B \oplus C) = (A \otimes B) \oplus (A \otimes C)$,

Idempotency: $A \oplus A = A$,

Distributive with respect to right multiplication: $(A \oplus B) \otimes C = (A \otimes C) \oplus (B \otimes C)$.

To complement the algebraic model, this study also uses Petri nets, which are widely applied to represent discrete event systems (Subiono, 2015). In Petri nets, the relationship between events is described using transitions and places. A transition occurs when all input places are marked, representing a change of state. A Petri net is formally defined as a 4-tuple (P, T, A, ω), where:

- P is a finite set of places $\{p_1, p_2, \ldots, p_n\}$,
- T is a finite set of transitions $\{t_1, t_2, \ldots, t_m\}$,
- A is a set of directed arcs such that $A \subseteq (P \times T) \cup (T \times P)$,
- ω is a weight function mapping each arc to a positive integer.

Although P and T are typically finite, they can also be countably infinite depending on the complexity of the system. In most practical cases, finite models are sufficient to represent systems such as outpatient service queues in hospitals.

According to Sari and Asih (2015), patients often wait for more than one hour to receive internal medicine services, highlighting the need for mathematical approaches capable of evaluating such systems more efficiently. Unlike traditional models that rely on probability or simulation, Max-Plus algebra offers a deterministic method to analyze discrete dynamic systems. When integrated with Petri nets, this approach allows for a clearer and more detailed visualization of service processes, making it easier to identify and improve inefficiencies.

Several previous studies have applied Max-Plus algebra to various real-world problems. These include outpatient service modeling with Petri net representations (Hardiyanti, Yuniwati, & Yustita, 2017; Tutupary & Lesnussa, 2013; Paulina Maure et al., 2021), scheduling in craft production (Yahya, Nurwan, & Resmawan, 2022), and optimization of tofu production processes (Nawar, Rahakbauw, & Patty, 2023). A related study by Saumi and Amalia (2021) modeled queues in a hospital's cardiology department using a single-channel, single-phase FIFO system. Another by Alamsyah and Sari (2023) utilized the simplex method via POM software to optimize resource allocation and healthcare quality.

The novelty of this study lies in the specific outpatient workflow and service structure observed, starting from patient registration to prescription retrieval at the pharmacy in a private hospital in Yogyakarta. Therefore, the aim of this study is to develop a Max-Plus algebra and Petri net-based model for outpatient service queues in a hospital, providing an analytical framework to better understand and evaluate service efficiency.

METHODS

This study is a qualitative descriptive research aimed at modeling the outpatient service queue system in a private hospital in Yogyakarta using the Max-Plus algebra approach and Petri net representation. Data were collected through direct observation of the outpatient service process, covering the flow from patient registration to medication retrieval at the pharmacy. Observations involved recording the arrival times and service durations at each stage using a stopwatch and structured observation sheets.

The collected data were used to construct a service flowchart. Based on this flow, a mathematical model was developed in the form of a Max-Plus matrix, which was then visualized as a Petri net to represent the discrete transitions and conditions within the system.

Data analysis was conducted descriptively using Scilab software to simulate the system and examine its dynamics. The analysis focused on determining whether the system exhibits periodic behavior, which is assessed based on the presence or absence of an eigenvalue in the Max-Plus matrix model.

FINDING AND DISCUSSION

Steps or Algorithm Used

The initial step of this research involved collecting data through direct observation. The service times at each stage of the outpatient process were recorded using a stopwatch. Based on the recorded data, a flowchart was developed to visualize the sequence of services experienced by patients from entry to exit.

Before constructing the mathematical model, variables were defined to represent each service duration and the transition time between stages. Using these variables, Max-Plus algebraic expressions were formulated to represent the relationships between service stages. These expressions were then used to build the corresponding Max-Plus matrix model.

The resulting matrix was then analyzed using Scilab software to calculate the eigenvalue of the system. The result of this calculation served as the basis for concluding the system's periodicity, service efficiency, and identifying possible improvements.

Outpatient Service Flowchart

The following figure presents the flowchart of the outpatient service process on the fourth floor of the private hospital in Yogyakarta. It includes all stages from patient admission, administrative procedures, consultation in examination rooms, to prescription collection and patient discharge.

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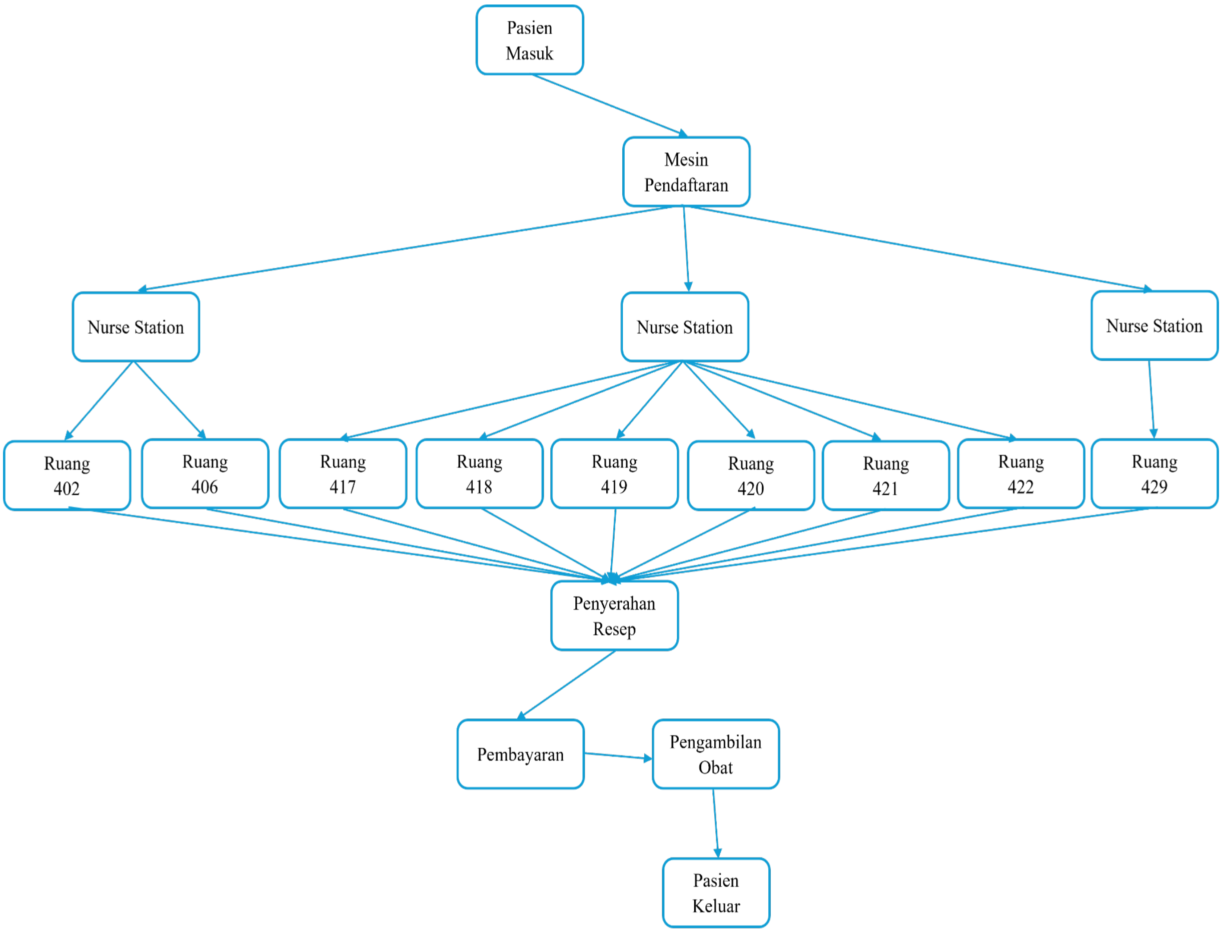

Figure 1. Outpatient Service Flowchart – 4th Floor

Max-Plus Algebra Model of the Outpatient Service Queue System in a Private Hospital in Yogyakarta

The variable descriptions used are as follows:

a(i) = patient arrival time on the 4th floor at the i-th
b(i) = time to start using the registration machine at the i-th
c(i) = time to finish using the registration machine at the i-th
d(i) = time to start service at nurse station 1 at the i-th
e(i) = time to finish service at nurse station 1 at the i-th
f(i) = time to start service at nurse station 2 at the i-th
g(i) = time to finish service at nurse station 2 at the i-th
h(i) = time to start service at nurse station 3 at the i-th
i(i) = time to finish service at nurse station 3 at the i-th
j(i) = time to start outpatient service in room 402 at the i-th

k(i) = time to finish outpatient service in room 402 at the i-th
l(i) = time to start outpatient service in room 406 at the i-th
m(i) = time to finish outpatient service in room 406 at the i-th
n(i) = time to start outpatient service in room 417 at the i-th
o(i) = time to finish outpatient service in room 417 at the i-th
p(i) = time to start outpatient service in room 418 at the i-th
q(i) = time to finish outpatient service in room 418 at the i-th
r(i) = time to start outpatient service in room 419 at the i-th
s(i) = time to finish outpatient service in room 419 at the i-th
t(i) = time to start outpatient service in room 420 at the i-th
u(i) = time to finish outpatient service in room 420 at the i-th
v(i) = time to start outpatient service in room 421 at the i-th
w(i) = time to finish outpatient service in room 421 at the i-th
x(i) = time to start outpatient service in room 422 at the i-th
y(i) = time to finish outpatient service in room 422 at the i-th
z(i) = time to start outpatient service in room 429 at the i-th
aa(i) = time to finish outpatient service in room 429 at the i-th
ab(i) = time to start prescription handover at the i-th
ac(i) = time to finish prescription handover at the i-th
ad(i) = time to start payment at the i-th
ae(i) = time to finish payment at the i-th
af(i) = time to start medication retrieval at the i-th
ag(i) = time to finish medication retrieval at the i-th

Va = average duration of registration machine service (in minutes)
Vb = average walking time from registration to nurse station (in minutes)
Vc = average nurse station service time (in minutes)
Vd = average walking time to consultation room (in minutes)
Ve = average consultation time with doctor (in minutes)
Vf = average walking time from doctor to prescription
Vg = average time for prescription handover (in minutes)
Vh = average payment time (in minutes)
Vi = average waiting time for medicine (in minutes)
Vj = average medication retrieval time (in minutes)

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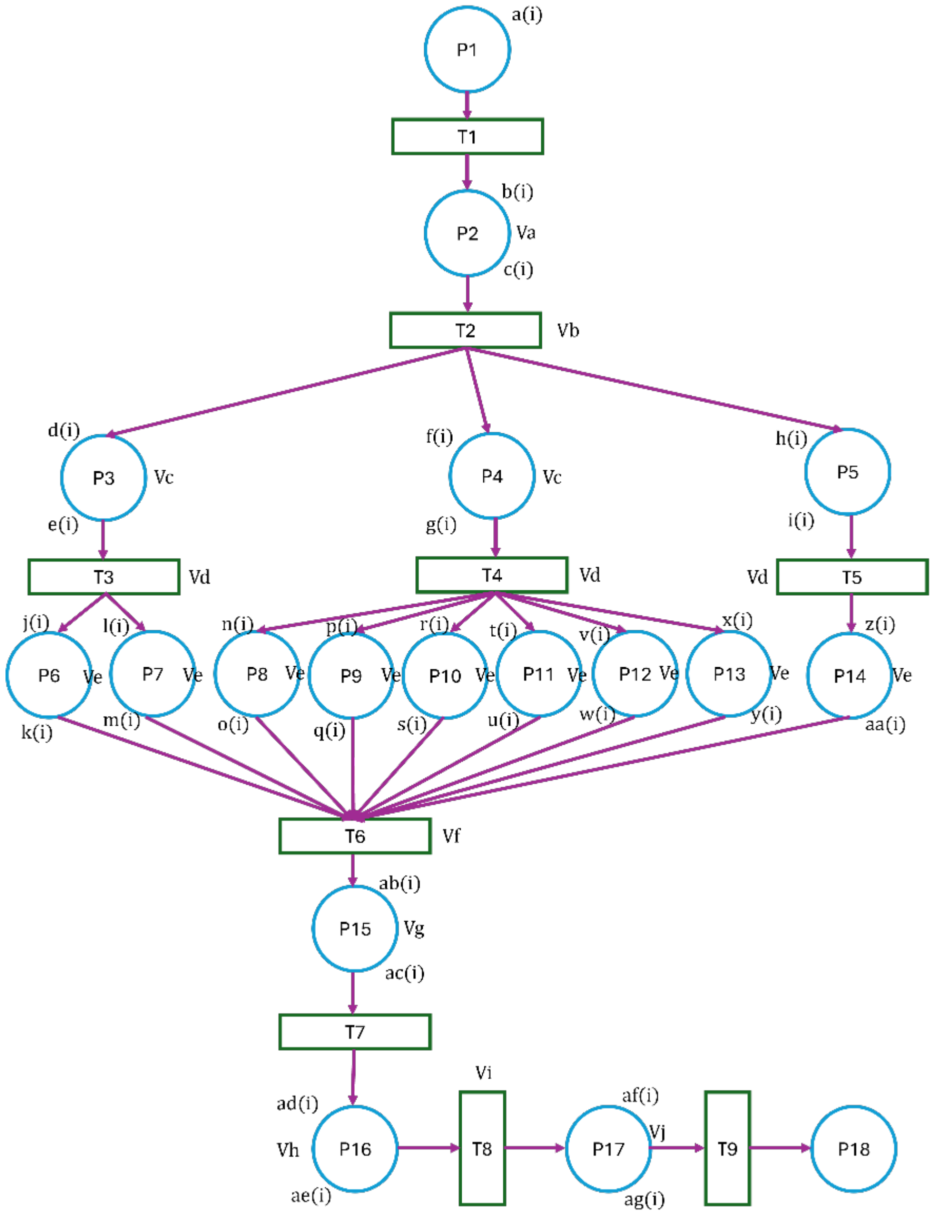

Figure 2. Petri Net of the Outpatient Queue System on the 4th Floor

Table 1. Average Service Time at Each Stage

Variable Code	Description of Service Time	Average (minutes)
Va	Registration machine service time	0.23
Vb	Walking time to nurse station	1.00
Vc	Nurse station service time	2.267
Vd	Walking time to doctor's room	1.00
Ve	Doctor consultation time	11.583

Variable Code	Description of Service Time	Average (minutes)
Vf	Walking time to prescription handover	1.00
Vg	Prescription handover time	0.80
Vh	Payment time	1.15
Vi	Waiting time for medication	28.083
Vj	Medication retrieval time	2.05

There are several assumptions used in this model, including: patients arrive at fixed intervals, each stage has a fixed average service time as listed in the table, all patients are served based on the order of arrival, and each service stage has a fixed number of staff.

$$
\begin{aligned}
a(i) &= a(i)\\
b(i) &= a(i) \oplus c(i-1)\\
c(i) &= b(i) \otimes Va\\
&= (a(i) \otimes 0{,}23) \oplus (c(i-1) \otimes 0{,}23)\\
d(i) &= (c(i) \otimes Vb) \oplus d(i-1)\\
&= (a(i) \otimes 1{,}23) \oplus (c(i-1) \otimes 1{,}23) \oplus d(i-1)\\
e(i) &= d(i) \otimes Vc\\
&= (a(i) \otimes 3{,}497) \oplus (c(i-1) \otimes 3{,}497) \oplus (d(i-1) \otimes 2{,}267)\\
f(i) &= (c(i) \otimes Vb) \oplus f(i-1)\\
&= (a(i) \otimes 1{,}23) \oplus (c(i-1) \otimes 1{,}23) \oplus f(i-1)\\
g(i) &= f(i) \otimes Vc\\
&= (a(i) \otimes 3{,}497) \oplus (c(i-1) \otimes 3{,}497) \oplus (f(i-1) \otimes 2{,}267)\\
h(i) &= (c(i) \otimes Vb) \oplus h(i-1)\\
&= (a(i) \otimes 1{,}23) \oplus (c(i-1) \otimes 1{,}23) \oplus h(i-1)\\
i(i) &= h(i) \otimes Vc\\
&= (a(i) \otimes 3{,}497) \oplus (c(i-1) \otimes 3{,}497) \oplus (h(i-1) \otimes 2{,}267)\\
j(i) &= (e(i) \otimes Vd) \oplus j(i-1)\\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (d(i-1) \otimes 3{,}267) \oplus j(i-1)\\
k(i) &= j(i) \otimes Ve\\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (d(i-1) \otimes 14{,}85) \oplus (j(i-1) \\
&\qquad \otimes 11{,}583)\\
l(i) &= (e(i) \otimes Vd) \oplus l(i-1)\\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (d(i-1) \otimes 3{,}267) \oplus l(i-1)\\
m(i) &= l(i) \otimes Ve\\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (d(i-1) \otimes 14{,}85) \oplus (l(i-1) \\
&\qquad \otimes 11{,}583)\\
n(i) &= (g(i) \otimes Vd) \oplus n(i-1)
\end{aligned}
$$

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$$= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus n(i-1)$$

$$\begin{aligned}
o(i) &= n(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (n(i-1) \\
&\qquad \otimes 11{,}583) \\
p(i) &= (g(i) \otimes Vd) \oplus p(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus p(i-1) \\
q(i) &= p(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (p(i-1) \\
&\qquad \otimes 11{,}583) \\
r(i) &= (g(i) \otimes Vd) \oplus r(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus r(i-1) \\
s(i) &= r(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (r(i-1) \\
&\qquad \otimes 11{,}583) \\
t(i) &= (g(i) \otimes Vd) \oplus t(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus t(i-1) \\
u(i) &= t(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (t(i-1) \\
&\qquad \otimes 11{,}583) \\
v(i) &= (g(i) \otimes Vd) \oplus v(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus v(i-1) \\
w(i) &= v(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (v(i-1) \\
&\qquad \otimes 11{,}583) \\
x(i) &= (g(i) \otimes Vd) \oplus x(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (f(i-1) \otimes 3{,}267) \oplus x(i-1) \\
y(i) &= x(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (f(i-1) \otimes 14{,}85) \oplus (x(i-1) \\
&\qquad \otimes 11{,}583) \\
z(i) &= (h(i) \otimes Vd) \oplus z(i-1) \\
&= (a(i) \otimes 4{,}497) \oplus (c(i-1) \otimes 4{,}497) \oplus (h(i-1) \otimes 3{,}267) \oplus z(i-1) \\
aa(i) &= z(i) \otimes Ve \\
&= (a(i) \otimes 16{,}08) \oplus (c(i-1) \otimes 16{,}08) \oplus (h(i-1) \otimes 14{,}85) \oplus (z(i-1) \\
&\qquad \otimes 11{,}583 \\
ab(i) &= \Big(\big(k(i) \oplus m(i) \oplus o(i) \oplus q(i) \oplus s(i) \oplus u(i) \oplus w(i) \oplus y(i) \oplus aa(i)\big) \otimes Vf\Big) \\
&\qquad \oplus ab(i-1)
\end{aligned}$$

$$
\begin{aligned}
&= (a(i) \otimes 17{,}08) \oplus (c(i-1) \otimes 17{,}08) \oplus (d(i-1) \otimes 15{,}85) \\
&\quad \oplus (f(i-1) \otimes 15{,}85) \oplus (h(i-1) \otimes 15{,}85) \\
&\quad \oplus (j(i-1) \otimes 12{,}582) \oplus (l(i-1) \otimes 12{,}582) \\
&\quad \oplus (n(i-1) \otimes 12{,}582) \oplus (p(i-1) \otimes 12{,}582) \\
&\quad \oplus (r(i-1) \otimes 12{,}582) \oplus (t(i-1) \otimes 12{,}582) \\
&\quad \oplus (v(i-1) \otimes 12{,}582) \oplus (x(i-1) \otimes 12{,}582) \\
&\quad \oplus (z(i-1) \otimes 12{,}582) \oplus ab(i-1)
\end{aligned}
$$

$$
\begin{aligned}
ac(i) &= ab(i) \otimes Vg \\
&= (a(i) \otimes 17{,}88) \oplus (c(i-1) \otimes 17{,}88) \oplus (d(i-1) \otimes 16{,}65) \\
&\quad \oplus (f(i-1) \otimes 16{,}65) \oplus (h(i-1) \otimes 16{,}65) \\
&\quad \oplus (j(i-1) \otimes 13{,}382) \oplus (l(i-1) \otimes 13{,}382) \\
&\quad \oplus (n(i-1) \otimes 13{,}382) \oplus (p(i-1) \otimes 13{,}382) \\
&\quad \oplus (r(i-1) \otimes 13{,}382) \oplus (t(i-1) \otimes 13{,}382) \\
&\quad \oplus (v(i-1) \otimes 13{,}382) \oplus (x(i-1) \otimes 13{,}382) \\
&\quad \oplus (z(i-1) \otimes 13{,}382) \oplus (ab(i-1) \otimes 0{,}8)
\end{aligned}
$$

$$
\begin{aligned}
ad(i) &= ac(i) \oplus ad(i-1) \\
&= (a(i) \otimes 17{,}88) \oplus (c(i-1) \otimes 17{,}88) \oplus (d(i-1) \otimes 16{,}65) \\
&\quad \oplus (f(i-1) \otimes 16{,}65) \oplus (h(i-1) \otimes 16{,}65) \\
&\quad \oplus (j(i-1) \otimes 13{,}382) \oplus (l(i-1) \otimes 13{,}382) \\
&\quad \oplus (n(i-1) \otimes 13{,}382) \oplus (p(i-1) \otimes 13{,}382) \\
&\quad \oplus (r(i-1) \otimes 13{,}382) \oplus (t(i-1) \otimes 13{,}382) \\
&\quad \oplus (v(i-1) \otimes 13{,}382) \oplus (x(i-1) \otimes 13{,}382) \\
&\quad \oplus (z(i-1) \otimes 13{,}382) \oplus (ab(i-1) \otimes 0{,}8) \oplus ad(i-1)
\end{aligned}
$$

$$
\begin{aligned}
ae(i) &= ad(i) \otimes Vh \\
&= (a(i) \otimes 19{,}03) \oplus (c(i-1) \otimes 19{,}03) \oplus (d(i-1) \otimes 17{,}8) \\
&\quad \oplus (f(i-1) \otimes 17{,}8) \oplus (h(i-1) \otimes 17{,}8) \oplus (j(i-1) \otimes 14{,}532) \\
&\quad \oplus (l(i-1) \otimes 14{,}532) \oplus (n(i-1) \otimes 14{,}532) \\
&\quad \oplus (p(i-1) \otimes 14{,}532) \oplus (r(i-1) \otimes 14{,}532) \\
&\quad \oplus (t(i-1) \otimes 14{,}532) \oplus (v(i-1) \otimes 14{,}532) \\
&\quad \oplus (x(i-1) \otimes 14{,}532) \oplus (z(i-1) \otimes 14{,}532) \oplus (ab(i-1) \\
&\quad \otimes 1{,}95) \oplus (ad(i-1) \otimes 1{,}15)
\end{aligned}
$$

$$
\begin{aligned}
af(i) &= (ae(i) \otimes Vi) \oplus (af(i-1) \otimes Vi) \\
&= (a(i) \otimes 47{,}113) \oplus (c(i-1) \otimes 47{,}113) \oplus (d(i-1) \otimes 45{,}883) \\
&\quad \oplus (f(i-1) \otimes 45{,}883) \oplus (h(i-1) \otimes 45{,}883) \\
&\quad \oplus (j(i-1) \otimes 42{,}615) \oplus (l(i-1) \otimes 42{,}615) \\
&\quad \oplus (n(i-1) \otimes 42{,}615) \oplus (p(i-1) \otimes 42{,}615) \\
&\quad \oplus (r(i-1) \otimes 42{,}615) \oplus (t(i-1) \otimes 42{,}615) \\
&\quad \oplus (v(i-1) \otimes 42{,}615) \oplus (x(i-1) \otimes 42{,}615) \\
&\quad \oplus (z(i-1) \otimes 42{,}615) \oplus (ab(i-1) \otimes 30{,}033) \oplus (ad(i-1) \\
&\quad \otimes 29{,}233) \oplus (af(i-1) \otimes 28{,}083)
\end{aligned}
$$

$$ag(i) = af(i) \otimes Vj$$

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$$
\begin{aligned}
= (a(i) \otimes 49{,}163) \oplus (c(i-1) \otimes 49{,}163) \oplus (d(i-1) \otimes 47{,}933) \\
\oplus (f(i-1) \otimes 47{,}933) \oplus (h(i-1) \otimes 47{,}933) \oplus (j(i-1) \otimes 44{,}665) \\
\oplus (l(i-1) \otimes 44{,}665) \oplus (n(i-1) \otimes 44{,}665) \oplus (p(i-1) \otimes 44{,}665) \\
\oplus (r(i-1) \otimes 44{,}665) \oplus (t(i-1) \otimes 44{,}665) \oplus (v(i-1) \otimes 44{,}665) \\
\oplus (x(i-1) \otimes 44{,}665) \oplus (z(i-1) \otimes 44{,}665) \oplus (ab(i-1) \otimes 32{,}083) \\
\oplus (ad(i-1) \otimes 31{,}283) \oplus (af(i-1) \otimes 30{,}133)
\end{aligned}
$$

Next, the model was formulated into the following Max-Plus algebra matrix.

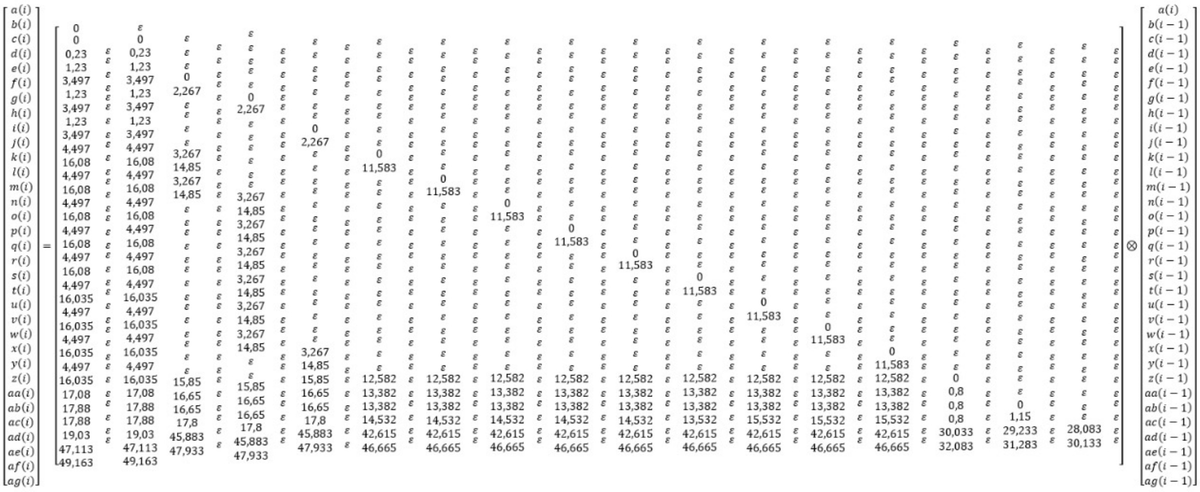

Figure 3. Max-Plus algebra matrix.

Based on the calculations, the Max-Plus algebra matrix was obtained from modeling the outpatient service queue system at a private hospital in Yogyakarta using Max-Plus Algebra. This model represents a theoretical representation.

This matrix is then used to analyze the queue system by simulating the system and calculating its eigenvalue. The simulation is useful for estimating the total time a patient needs to complete the entire outpatient service process. Meanwhile, the eigenvalue helps determine whether there is a recurring pattern or stability in the service cycle of outpatient care at the hospital. The analysis was conducted using Scilab 5.5.2 software.

Based on the simulation results, it was observed that the increase in values from left to right indicates that the cumulative service time increases over time. Rows with higher values may indicate stages that take longer or experience queue buildup. The maximum recorded value is 301.91 minutes or approximately 5 hours, meaning that the total time required to complete all outpatient processes for the last patient in the worst-case scenario is 5 hours. This is a relatively long waiting time, which could lead to patient dissatisfaction.

1) To address this issue, we offer the following recommendations for the hospital: Increase the number of staff at stages with the longest service times,
2) Optimize the overall service flow, and

3) Implement a scheduling system to reduce patient arrival surges.

The following is the result of the simulation using Scilab 5.5.2:

```
-->[X]=maxplussys(A,[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0], 10)
 X  =

    0.    0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
    0.    0.        0.23      0.46      0.69      0.92      1.15      1.38      1.61      1.84      2.07
    0.    0.23      0.46      0.69      0.92      1.15      1.38      1.61      1.84      2.07      2.3
    0.    1.23      1.46      1.69      1.92      2.15      2.38      2.61      2.84      3.07      3.3
    0.    3.497     3.727     3.957     4.187     4.417     4.647     4.877     5.107     5.337     5.567
    0.    1.23      1.46      1.69      1.92      2.15      2.38      2.61      2.84      3.07      3.3
    0.    3.497     3.727     3.957     4.187     4.417     4.647     4.877     5.107     5.337     5.567
    0.    1.23      1.46      1.69      1.92      2.15      2.38      2.61      2.84      3.07      3.3
    0.    3.497     3.727     3.957     4.187     4.417     4.647     4.877     5.107     5.337     5.567
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.08     16.31     16.54     16.77     17.       17.23     17.46     17.69     17.92     18.15
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.08     16.31     16.54     16.77     17.       17.23     17.46     17.69     17.92     18.15
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.08     16.31     16.54     16.77     17.       17.23     17.46     17.69     17.92     18.15
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.08     16.31     16.54     16.77     17.       17.23     17.46     17.69     17.92     18.15
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.08     16.31     16.54     16.77     17.       17.23     17.46     17.69     17.92     18.15
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.035    16.265    16.495    16.725    16.955    17.185    17.415    17.645    17.875    18.105
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.035    16.265    16.495    16.725    16.955    17.185    17.415    17.645    17.875    18.105
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.035    16.265    16.495    16.725    16.955    17.185    17.415    17.645    17.875    18.105
    0.    4.497     4.727     4.957     5.187     5.417     5.647     5.877     6.107     6.337     6.567
    0.    16.035    16.265    16.495    16.725    16.955    17.185    17.415    17.645    17.875    18.105
    0.    17.08     17.31     17.54     17.77     18.       18.23     18.46     18.69     18.92     19.15
    0.    17.88     18.11     18.34     18.57     18.8      19.03     19.26     19.49     19.72     19.95
    0.    17.88     18.11     18.34     18.57     18.8      19.03     19.26     19.49     19.72     19.95
    0.    19.03     20.029    20.259    20.489    20.719    20.949    21.179    21.409    21.639    21.869
    0.    47.113    75.196    103.279   131.362   159.445   187.528   215.611   243.694   271.777   299.86
    0.    49.163    77.246    105.329   133.412   161.495   189.578   217.661   245.744   273.827   301.91
```

Figure 3. Queue System Simulation Results

Additionally, by observing the bottom row, it can be concluded that the time difference between patients from registration to payment is approximately 30 minutes. Based on the developed model, several optimization actions can be taken, such as adding staff in the medication retrieval stage (which has the highest average service time) and adopting an electronic queue system to minimize administrative delays.

This study differs from the findings of Hardiyanti, Yuniwati, and Divi Yustita (2017), who used Max-Plus algebra to model outpatient services at Al Huda Genteng Hospital. Their model showed shorter average waiting times due to a focus on prioritized patients.

Next, an analysis was conducted to calculate the eigenvalue. According to the results from Scilab, the matrix does not have an eigenvalue. This indicates irregular behavior in waiting times and arrival patterns of each patient. The absence of an eigenvalue suggests that each stage in the outpatient service process is non-periodic.

The model developed in this study has significant potential to support hospital management in improving service efficiency. With this model, hospital administrators can monitor service bottlenecks and simulate outcomes before

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implementing operational changes, such as increasing staff or optimizing workflow. Moreover, the model enables more precise resource planning based on daily patient volume. Thus, it functions not only as an analytical tool but also as a practical guide for strategic decision-making.

This model also holds potential for broader applications beyond outpatient services. For instance, it could be applied to radiology services, where long waits are common due to limited equipment and personnel. Similarly, the pharmacy's medication pickup queues could be optimized using this approach. Beyond healthcare, the model is relevant to queue systems in transportation (e.g., airport check-ins) and logistics (e.g., warehouse distribution). With its flexibility, the model can be adapted to various operational settings with similar queuing patterns.

CONCLUSIONS AND RECOMMENDATIONS

Based on the results and discussion, the outpatient queue system at a private hospital in Yogyakarta can be modeled using Max-Plus Algebra. The modeling process resulted in a Max-Plus algebra matrix that represents the flow of services mathematically. Prior to matrix construction, a Petri net was developed to visualize the service sequence and support the modeling process.

This study was limited to the formulation of the matrix model. The resulting Max-Plus matrix can be further utilized for analyzing the outpatient queue system in more depth, particularly in evaluating service performance and identifying potential delays. Follow-up research may focus on applying this model to conduct simulations, optimize the queue system, or support hospital management in improving operational efficiency.

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