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THE LOCATING-CHROMATIC NUMBER FOR CERTAIN OPERATION OF PIZZA GRAPHS

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ABSTRACT This study investigates the location chromatic number resulting from specific operations on pizza graphs. The location chromatic number of a graph extends the concept of vertex coloring and graph partition dimension. This research aims to deepen the understanding of how certain operations influence the location chromatic number of pizza graphs. By conducting rigorous mathematical analysis, this study demonstrates the effects of particular operations on the chromatic properties of these graphs. The results provide valuable insights into the characteristics of pizza graphs, which are known for their complex and irregular structures. These findings contribute to the broader development of graph theory, particularly in understanding modified graph structures. Additionally, the research highlights the potential applications of these concepts in various scientific and mathematical domains. This study serves as a foundation for future exploration of chromatic properties in more complex and diverse graph structures.

Keywords: location chromatic number, pizza graph, graph theory, graph operations

ABSTRAK Penelitian ini menyelidiki bilangan kromatik lokasi yang dihasilkan dari operasi tertentu pada graf pizza. Bilangan kromatik lokasi suatu graf merupakan perluasan dari konsep pewarnaan titik dan dimensi partisi graf. Penelitian ini bertujuan untuk memperdalam pemahaman tentang bagaimana operasi tertentu memengaruhi bilangan kromatik lokasi pada graf pizza. Melalui analisis matematis yang ketat, penelitian ini menunjukkan pengaruh operasi tertentu terhadap sifat kromatik dari graf tersebut. Hasilnya memberikan wawasan berharga mengenai karakteristik graf pizza yang dikenal memiliki struktur kompleks dan tidak teratur. Temuan ini berkontribusi pada pengembangan teori graf secara lebih luas, khususnya dalam memahami struktur graf yang telah dimodifikasi. Selain itu, penelitian ini menyoroti potensi aplikasi dari konsep-konsep ini dalam berbagai domain ilmiah dan matematis. Penelitian ini menjadi dasar untuk eksplorasi lebih lanjut mengenai sifat kromatik pada struktur graf yang lebih kompleks dan beragam.

Keywords: bilangan kromatik lokasi, graf pizza, teori graf, operasi graf

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INTRODUCTION

In 2000 (Chartrand et al. 2000) explored the partition dimension of connected graphs in their research, proposing a novel method to tackle the problem of determining the metric dimension in graphs. The concept of metric dimensions is practically important for guiding a robot, modeled as a graph, during navigation. (Saenpholphat & Zhang, 2004), as well as tackling the challenge of classifying chemical data, which requires finding an efficient way to represent a set of chemical compounds and ensuring that different compounds have unique representations (Johnson. 1993). The concept of locating-chromatic numbers is a fundamental aspect of graph theory. Introduced by (Chartrand et al., 2002) It involves assigning colors to the vertices of a graph so that the colors of its neighbors can uniquely determine each vertex. This concept has applications in various fields, from robotics to chemical data classification.

The locating-chromatic number of a graph was introduced by (Chartrand et al., 2002) which has become one of the important topics in graph theory. This concept combines two important ideas in graph theory : the proper coloring of vertices and the partition dimension of a graph. This concept focuses on determining the minimum number of colors required to color the vertices of a graph such that each vertex can be uniquely identified based on the colors of its neighbors. This research opened new horizons in graph studies, particularly related to applications in coding, networks, and various other combinatorial problems. Next, the research conducted by (Irawan et al. 2021) identify a method for calculating the locating-chromatic number of an origami graph and its subdivision, focusing along an outer edge. Furthermore, the locating chromatic number has also been determined for certain barbell operation and their subdivisions (Irawan et al. 2021).

Further research (Prawinasti et al., 2021) have calculated the locating-chromatic number for the path split graph. Next, (Rahmatalia et al., 2022) have identified the locating-chromatic number for the cycle's split graph, the locating-chromatic number of a helm graph determinded by (Lessya et al., 2024) and (Zulkarnain et al., 2024) find the locating chromatic number of the disjoint union of Buckminsterfullerene graphs . Asmiati et al. discovered the exploration of the locating-chromatic number in certain operation involving origami graphs (Asmiati et al., 2023) and locating-chromatic number for barbell operation of shadow path graphs (Asmiati et al., 2021). Next, Surbaki et al. (Surbakti et al., 2023) determined the locating-chromatic number of pizza graph, the result obtained is 4 for $s = 3$ and s for $s \geq 4$. Until now, there has been no algorithm to determine the locating-chromatic number of any graph. Therefore, an in-depth study is needed on the locating-chromatic number of the resulting graph operation. Our exploration centers on a unique class of graphs arising from the amalgamation of certain operation and pizza graphs, introducing a novel perspective to the existing body of literature. The primary objective of this study is to determine the locating-chromatic

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number for certain operation applied to pizza graphs. The certain operation of a pizza graphs $CO_{P_{Z_s}}$ is a graph obtained by connecting two pizza graphs P_{Z_s} with a path along all outer edges $z_s z_{s+1}$.

PRELIMINARIES

In this section, we will concisely review several key definitions and theorems fundamental to understanding our research context. These concepts form the theoretical basis and will be referenced throughout the discussion of our main findings.

The locating-chromatic number for a graph is defined as follows:

Definition 1. (Chartrand et al. 2000) Let $D = (V, E)$ be a graph that is connected and c be a proper r-coloring of D with color $\{1, 2, \ldots, r\}$. Let $\Pi = \{T_1, T_2, \ldots, T_r\}$ be a partition of $V(B)$ which is induced by coloring c. The color code $c_\Pi(u)$ of u is the ordered r-tuple $(d(u, T_1), d(u, T_2), \ldots, d(u, T_r))$ where $d(u, T_j) = \min\{d(u, x) : x \in T_j\}$ for any $j \in \{1, 2, \ldots, r\}$. If all distinct vertices of D have distinct color codes, then e is called k-locating coloring of D. The locating chromatic number, denoted by $\chi_L(D)$, is the smallest r such that D has a locating r-coloring.

The next pizza graph definition is taken from Nabila and Salman :

Definition 2. (Nabila and Salman, 2015) A pizza graph P_{Z_s} is a graph with $V(P_{Z_s}) = \{x, y_j, z_j : j \in \{1, \ldots, s\}\}$ and $E(P_{Z_s}) = \{xy_j, y_j z_j : j \in \{1, \ldots, s\}\} \cup \{z_j z_{j+1} : j \in \{1, \ldots, s-1\}\} \cup \{z_s z_1\}$.

An example of a pizza graph can be seen in the following Figure 1.

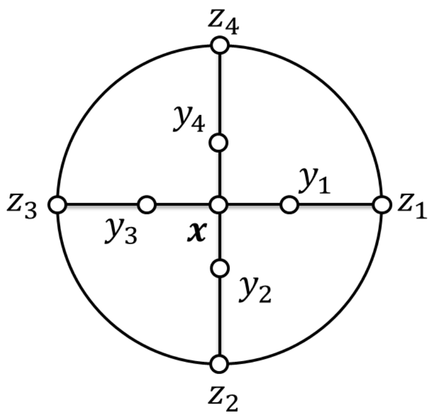

Figure 1. A pizza graph for $s = 4$

Next, a certain operation of the pizza graph will be defined as follows:

Definition 3.The certain operation of a pizza graphs $CO_{P_{Z_s}}$ is a graph obtained by connecting two pizza graphs P_{Z_s} with a path along all outer edges $z_s z_{s+1}$. A certain operation of pizza graphs $CO_{P_{Z_s}}$ is a graph with $V\left(CO_{P_{Z_s}}\right) = \{x_1, x_2, y_j, y_{s+j}, z_j, z_{s+j} : j \in \{1, \dots, s\}\}$ and $E\left(CO_{P_{Z_s}}\right) = \{x_1 y_j, x_2 y_{s+j}, y_j z_j, y_{s+j}, z_{s+j} : j \in \{1, \dots, s\}\} \cup \{z_j z_{j+1}, z_{s+j} z_{s+j+1} \ : j \in \{1, \dots, s-1\}\} \cup \{z_s z_1, z_{2s} z_{s+1}\}$.

The following theorems are essential for determining the lower bound of a graphs locating chromatic number. The collection of neighboring vertices of a vertex s in B is represented by $N(s)$.

Theorem 1. (Chartrand et al., 2002) Suppose c represents a locating coloring in a connected graph B. If r and s are distinct vertices of B such that $d(r,z) = d(s,z)$ for all $z \in V\,(B) - \{r,s\}$, then $c(r) \neq c(s)$. In particular, if r and s are non-adjacent vertices such that $N(r) \neq N(s)$, then $c(r) \neq c(s)$.

Theorem 2. (Surbakti et al., 2023) The locating-chromatic number of the pizza graph is 4 for s = 3 and s for s ≥ 4.

FINDING AND DISCUSSION

The next section explains the locating chromatic number for certain operation on pizza graphs.

Theorem 3. Let $CO_{P_{Z_s}}$ certain operation of pizza graphs for $m \geq 3$. Then :

$$\chi_L\left(CO_{P_{Z_s}}\right) = \begin{cases} 5, & \text{for } s = 3 \\ s+1 & \text{otherwise.} \end{cases}$$

Proof. A certain operation of pizza graphs $CO_{P_{Z_s}}$ is a graph with $V\left(CO_{P_{Z_s}}\right) = \left\{x_1, x_2, y_j, y_{s+j}, z_j, z_{s+j} : j \in \{1, \dots, s\}\right\}$ and $E\left(CO_{P_{Z_s}}\right) = \left\{x_1 y_j, x_2 y_{s+j}, y_j z_j, y_{s+j}, z_{s+j} : j \in \{1, \dots, s\}\right\} \cup \left\{z_j z_{j+1}, z_{s+j} z_{s+j+1} : j \in \{1, \dots, s-1\}\right\} \cup \{z_s z_1, z_{2s} z_{s+1}\}$. Let's distinguish between two cases to establish the upper bound for the locating-chromatic number of certain operation on pizza graphs.

Case 1. For s = 3

First, we will establish the lower bound of $\chi_L\left(CO_{P_{Z_3}}\right)$. Considering that the certain operation of pizza graphs $CO_{P_{Z_3}}$ contains two identical copies of the pizza graph P_{Z_3}, as per Theorem 2 we have the locating-chromatic number of the pizza graph is 4. Next, suppose that t is a locating coloring using 4 colors. It can be easily shown that for any certain operation on the pizza graph $CO_{P_{Z_3}}$ there must be two vertices with the same color code, which leads to a contradiction. Thus, we have that $\chi_L\left(CO_{P_{Z_3}}\right) \geq 5$.

Following that, we established the upper bound of $\chi_L\left(CO_{P_{Z_3}}\right) \leq 5$. To show that$\chi_L\left(CO_{P_{Z_3}}\right) \leq 5$, we apply coloring t, utilizing 5 colors in the following manner: $T_1 = \{y_3, y_6, z_1, z_4\}$; $T_2 = \{y_1, y_4, z_2, z_5\}$; $T_3 = \{y_2, y_5, z_3, z_6\}$; $T_4 = \{x_2\}$; $T_5 = \{x_1\}$. With the application of coloring t, we derive the color codes for $V\ (CO_{P_{Z_3}})$ as follows: $t_\Pi(x_1) = (1,1,1,5,0)$; $t_\Pi(x_2) = (1,1,1,0,5)$; $t_\Pi(y_1) = (1,0,2,5,1)$; $t_\Pi(y_2) = (2,1,0,5,1)$; $t_\Pi(y_3) = (0,2,1,4,1)$; $t_\Pi(y_4) = (1,0,2,1,4)$; $t_\Pi(y_5) = (2,1,0,1,4)$; $t_\Pi(y_6) = (0,2,1,1,4)$; $t_\Pi(z_1) = (0,1,1,4,2)$; $t_\Pi(z_2) = (1,0,1,4,2)$; $t_\Pi(z_3) = (1,1,0,3,2)$; $t_\Pi(z_4) = (0,1,1,2,3)$; $t_\Pi(z_5) = (1,0,1,2,3)$; $t_\Pi(z_6) = (1,1,0,2,3)$. Very clearly, the color codes of all vertices in $CO_{P_{Z_3}}$ are different, then t is a locating coloring. So $\chi_L\left(CO_{P_{Z_3}}\right) \leq 5$.

Case 2. For s ≥ 4

First, we will establish the lower bound of $\chi_L\left(CO_{P_{Z_s}}\right)$ for s ≥ 4. Considering that the specific barbell operation of pizza graphs COPzs contains two isomorphic copies of the pizza graph P_{Z_s} , as per Theorem 2 we have $\chi_L\left(CO_{P_{Z_s}}\right) \geq$ s, for s ≥ 4. Next, suppose that t is a locating coloring using m colors. It can be easily shown that for any certain operation on the pizza graph $CO_{P_{Z_s}}$ there must be two vertices with the same color code, which leads to a contradiction. Thus, we have that $\chi_L\left(CO_{P_{Z_s}}\right) \geq$ s+1. Subsequently, to show that s+1 serves as an upper bound for the locating-chromatic number in certain operation of the pizza graphCOPzs , it is enough to establish the existence of a locating coloring t : $V\ (CO_{P_{Z_s}}) \to \{1, 2, \ldots, s+1\}$. For s ≥ 4, we formulate the function t as follows :

$$t(x_1) = s + 1$$

$$t(x_2) = s$$

$$t(y_j) = \begin{cases} j+1, & \text{for } j \in \{1, \dots, s-2\} \\ 1, & \text{for } j \in \{s-1, \dots, s\} \end{cases}$$

$$t(y_{s+j}) = \begin{cases} j+2, & \text{for } j \in \{1, \dots, s-3\} \\ 1, & \text{for } j \in \{s-2, \dots, s-1\} \\ 2, & j = s \end{cases}$$

$$t(z_j) = j, \text{ for } j \in \{1, \dots, s\}$$

$$t(z_{s+j}) = \begin{cases} j+1, & \text{for } j \in \{1, \dots, s-1\} \\ 1, & \text{for } j = s \end{cases}$$

With the application of coloring t, we derive the color codes for $V\ (CO_{P_{Z_s}})$ as follows:

$$t_\Pi(x_1)=\begin{cases}0, & \text{for } (s+1)^{th} \text{ordinate}\\ 2, & \text{for } s^{th} \text{ ordinate}\\ 1, & \text{otherwise.}\end{cases}$$

$$t_\Pi(y_j)=\begin{cases}0, & \text{for } (j+1)^{th}\text{ordinate, } j\in\{1,\dots,s-2\}\\ & \text{for } 1^{st} \text{ ordinate, } j\in\{s-1,\dots,s\}\\ 1, & \text{for } j^{th} \text{ ordinate, } j\in\{1,\dots,s\}\\ & \text{for } (s+1)^{th}\text{ordinate, } j\in\{1,\dots,s\}\\ & \text{for } s^{th} \text{ ordinate, } j=s\\ & \text{for } 1^{st} \text{ ordinate, } j=1\\ 2, & \text{for } s^{th} \text{ ordinate, } j=1\\ & \text{for } s^{th} \text{ ordinate, } j=s-1\\ & \text{for } (j+2)^{th}\text{ordinate, } j\in\{2,\dots,s-3\}\\ & \text{for } 1^{st} \text{ ordinate, } j\in\{2,\dots,s-2\}\\ 3, & \text{otherwise.}\end{cases}$$

$$t_\Pi(z_j)=\begin{cases}0, & \text{for } j^{th}\text{ordinate, } j\in\{1,\dots,s\}\\ 1, & \text{for } (j+1)^{th}\text{ordinate, } j\in\{1,\dots,s-1\}\\ & \text{for } (j-1)^{th}\text{ordinate, } j\in\{2,\dots,s\}\\ & \text{for } 1^{st} \text{ ordinate, } j\in\{s-1,\dots,s\}\\ 2, & \text{for } (j+2)^{th} \text{ ordinate, } j\in\{1,\dots,s-2\}\\ & \text{for } (j-2)^{th} \text{ ordinate, } j\in\{s-1,\dots,s\},\ s\geq 5\\ & \text{for } (s+1)^{th}\text{ordinate, } j\in\{1,\dots,s\}\\ & \text{for } 2^{nd} \text{ ordinate, } j=s\\ 4, & \text{for } s^{th} \text{ ordinate, } j\in\{5,\dots,s-4\},\ s\geq 9\\ j, & \text{for } s^{th} \text{ ordinate, } j=1,2,3\\ s-j, & \text{for } s^{th}\text{ordinate, } j\in\{s-3,\dots,s-1\}\\ 3, & \text{otherwise.}\end{cases}$$

$$t_\Pi(x_2)=\begin{cases}0, & \text{for } (s+1)^{th}\text{ordinate}\\ 5, & \text{for } (s+1)^{th} \text{ ordinate}\\ 1, & \text{otherwise.}\end{cases}$$

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$$t_\Pi(y_{s+j}) = \begin{cases} 0, & \text{for } (j+2)^{th} \text{ordinate}, \ j \in \{1, \dots, s-3\} \\ & \text{for } 1^{st} \text{ ordinate}, j \in \{s-2, \dots, s-1\} \\ & \text{for } 2^{nd} \text{ ordinate}, j = s \\ 1, & \text{for } (j+1)^{th} \text{ordinate}, j \in \{1, \dots, s-1\} \\ & \text{for } s^{th} \text{ ordinate}, j \in \{1, \dots, s\} \\ & \text{for } 1^{st} \text{ ordinate}, j = s \\ 4, & \text{for } (s+1)^{th} \text{ordinate}, \ j \in \{1, \dots, s\} \\ 2, & \text{otherwise.} \end{cases}$$

$$t_\Pi(z_{s+j}) = \begin{cases} 0, & \text{for } (j+2)^{th} \text{ordinate}, \ j \in \{1, \dots, s-1\} \\ & \text{for } 1^{st} \text{ ordinate}, j = s \\ 1, & \text{for } (j+2)^{th} \text{ordinate}, \ j \in \{1, \dots, s-3\} \\ & \text{for } j^{th} \text{ordinate}, \ j \in \{2, \dots, s-1\} \\ & \text{for } s^{th} \text{ ordinate}, j \in \{s-2, \dots, s\} \\ & \text{for } 1^{st} \text{ ordinate}, j = 1 \\ 2, & \text{for } (j+3)^{th} \text{ ordinate}, j \in \{1, \dots, s-3\} \\ & \text{for } (j-2)^{th} \text{ ordinate}, j \in \{2, \dots, s-1\}, s \geq 5 \\ & \text{for } (s-1)^{th} \text{ordinate}, j = 1 \\ & \text{for } s^{th} \text{ ordinate}, j \in \{1, \dots, s-3\} \\ & \text{for } 1^{st} \text{ ordinate}, j = 1 \\ & \text{for } 1^{st} \text{ ordinate}, j = s-3, s \geq 5 \\ 3, & \text{otherwise.} \end{cases}$$

Clearly, the color codes of all vertices in $V\left(CO_{P_{Z_s}}\right)$ are different. Then the coloring t is a locating coloring. So $\chi_L\left(CO_{P_{Z_s}}\right) \leq s+1$. ■

In a previous study (Surbakti et al., 2023), the locating-chromatic number of a pizza graph P_{Z_s} was found to be 4 for $s = 3$ and s for $s \geq 4$. After performing a certain operation on the pizza graph $CO_{P_{Z_s}}$, the result was 5 for $s = 3$ and $s + 1$ for $s \geq 4$. This demonstrates how a certain operation affects the locating-chromatic number of a pizza graph.

Figure 2 below provides an example of $\chi_L\left(CO_{P_{Z_4}}\right) = 5$.

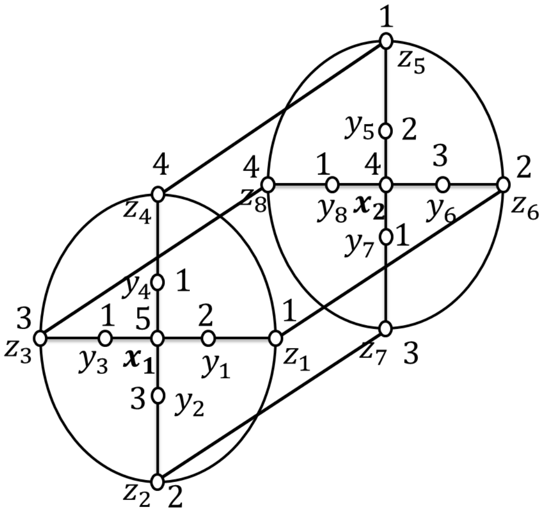

Figure 2. A minimum locating coloring of $CO_{P_{Z_4}}$.

CONCLUSIONS AND RECOMMENDATIONS

The conclusion reached from this discussion is $\chi_L\left(CO_{P_{Z_s}}\right) = \chi_L(P_{Z_s}) + 1$. This research analyzes how the locating-chromatic number changes as pizza graphs are modified with certain operation. For future research, the locating chromatic number of certain operation for n −pizza graphs can be determined.

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